SYNTHETIC JET FLOW CONTROL OF
TWO-DIMENSIONAL NACA 65(1)-412 AIRFOIL FLOW WITH
FINITE-TIME LYAPUNOV EXPONENT ANALYSIS OF
LAGRANGIAN COHERENT STRUCTURES

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Synthetic Jet Flow Control Of Two-Dimensional NACA 65(1)-412 Airfoil Flow With
Finite-Time Lyapunov Exponent Analysis Of Lagrangian Coherent Structures

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by
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DEDICATION

Dedicated to my family and friends, who motivate me for greatness.
Research is what I do when I don’t know what I’m doing.

– Wernher von Braun
ABSTRACT OF THE THESIS

Synthetic Jet Flow Control Of Two-Dimensional NACA 65(1)-412 Airfoil Flow With Finite-Time Lyapunov Exponent Analysis Of Lagrangian Coherent Structures

by

Peter Inuk Jeong

Master of Science in Aerospace Engineering
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Synthetic jet (SJ) control of a low-Reynolds number, unsteady, compressible, viscous flow over a NACA 65-(1)412 airfoil, typical for unmanned air vehicles and gas turbines, has been investigated computationally. A particular focus was placed in the development and control of Lagrangian Coherent Structures (LCS) and the associated Finite-Time Lyapunov Exponent (FTLE) fields. The FTLE fields quantitatively measure of the repulsion rate in forward-time and the attraction rate in backward-time, and provide a unique perspective on effective flow control.

A Discontinuous-Galerkin (DG) methods, high-fidelity Navier-Stokes solver performs direct numerical simulation (DNS) of the airfoil flow. Three SJ control strategies have been investigated: immediately downstream of flow separation, normal to the separated shear layer; near the leading edge, normal to the airfoil suction side; near the trailing edge, normal to the airfoil pressure side. A finite difference algorithm computes the FTLE from DNS velocity data.

A baseline flow without SJ control is compared to SJ actuated flows. The baseline flow forms a regular, time-periodic, asymmetric von Karman vortex street in the wake. The SJ downstream of flow separation increases recirculation region vorticity and reduces the effective angle of attack. This decreases the time-averaged lift by 2.98% and increases the time-averaged drag by 5.21%. The leading edge SJ produces small vortices that deflect the shear layer downwards, and decreases the effective angle of attack. This reduces the time-averaged lift by 1.80%, and the time-averaged drag by 1.84%. The trailing edge SJ produces perturbations that add to pressure side vortices without affecting global flow characteristics. The time-averaged lift decreases by 0.47%, and the time-averaged drag increases by 0.20%. For all SJ cases, the aerodynamic performance is much more dependent on changes to the pressure distribution than changes to the skin friction distribution.

No proposed SJ case improved aerodynamic performance. Some desirable SJ control effects were observed, which may be isolated in a future study by optimizing SJ parameters. Stably increasing recirculation region vorticity, and maintaining or increasing the effective angle of attack are desirable for lift increase, while deflecting the separated shear layer downward is desirable for drag reduction.
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CHAPTER 1
INTRODUCTION

Low-Reynolds number flows over an airfoil are frequently found in industrial applications of micro aerial vehicles (MAVs) and compressor blades. Flow separation from airfoil of such flows cause large pressure losses that deteriorate the performance of such machinery. Minimizing flow separation by delaying separation and/or accelerating flow reattachment broadens the operating envelope, extends the service life, reduces the fuel consumption and decrease emissions of industrial applications.

Synthetic jets (SJ) have received attention as a promising active flow control (AFC) strategy for control of vortex-dominated flows, and especially flows separating from aerodynamic surfaces [1–6]. Synthetic jet actuation uses only the periodic motion of a diaphragm or a piston to generate mass and momentum fluxes. The SJ is an appealing flow control device because, it is small enough to be implemented in aircraft flow situations [4], adaptable to various operating conditions [6], does not require an external mass source [2], and requires one to two orders of magnitude less momentum to produce equivalent control authority as steady blowing or suction devices [7]. Much of computational and experimental research of the SJ in literature report enhancement of airfoil performance by imposing SJ actuation near the separation point on the airfoil suction side [4–6]. Successful SJ control can delay flow separation [5], reduce separation length [2], increase maximum lift and stall angle [8], improve overall lift and lift-to drag ratio [9] and provide robust flow control by firmly attaching local flow to airfoil surface [4]. The flow physics of SJ forcing and the associated flow control authority depend on many parameters, including the location of the SJ relative to the separation point, the frequency of actuation, the jet flow angle relative to the free stream, and the ratio of SJ velocity to the free stream velocity. A combination of any of the aforementioned parameters can also alter the flow in an unexpected manner [1]. Therefore, the understanding of SJ flow control is far from complete. A bulk of published research on the SJ has focused on isolating different SJ operating parameters and their effects [2, 4, 6].

Zhang and Samtaney [6] conducted a direct numerical simulation (DNS) study of SJ frequency effects in control of flow over a NACA-0018 airfoil at chord Reynolds number $Re_c = 10^4$. The SJ was placed immediately upstream of the most upstream instantaneous separation point, and simulated at 3 different chord-based non-dimensional frequencies, $F^+ = fc/U_\infty = 0.5, 1,$ and 4, where $f$ is the dimensional frequency of the SJ, $c$ is the chord and $U_\infty$ is the free stream velocity. The SJ created vortices, that broke up and merged at
$F^+ = 0.5$, or remained discrete at $F^+ = 4$, as they advected downstream. At $F^+ = 1$, the vortex physics was at a transitional state between cases at $F^+ = 0.5$ and at $F^+ = 4$. Lift and lift-to-drag ratio increased for all SJ frequencies, and the highest performance improvements resulted at $F^+ = 1$. Dandois et al. [2] performed DNS and LES studies to study the frequency effects in SJ control of flow over a backward step at step height Reynolds number of $Re_h = 28275$. The SJ was simulated at separation length based non-dimensional frequencies $F^+ = \frac{fl_s}{U_\infty} = 0.5$, and 4, where $l_s$ is the separation length. At $F^+ = 0.5$, the SJ frequency was close to the natural vortex shedding frequency of the baseline flow and the SJ operated at a vorticity-dominated mode. At $F^+ = 4$, the SJ operated at an acoustic-dominated mode. SJ forcing near the natural vortex shedding frequency was favorable, as the separation length was reduced by 54% at $F^+ = 0.5$, but increased by 43% when $F^+ = 4$. Kim and Kim [4] performed a URANS study of different configurations of SJ control of flow over a NACA 23012 airfoil at chord Reynolds number $Re_c = 2.19 \times 10^6$. The SJs were actuated at velocity ratios of $U_{jet}/U_\infty = 1$, 2, and 3, and at non-dimensional frequencies of $F^+ = \frac{fl_c}{U_\infty} = 1$, 4, and 5, where $l_c$ is the distance between the trailing edge and the SJ. The SJ control resized separation vortices, improved stall characteristics and enhanced aerodynamic performance. At a low frequency range, the SJ created small vortices that penetrated the leading edge separation vortex and subsequently reduced the size of the separation vortex remarkably. At a high frequency range, the small vortices generated by the SJ did not penetrate the leading edge separation vortex, and the SJ firmly attached the local flow. SJ velocity and flow control effectiveness were proportional. The maximum lift was obtained when the SJ was actuated at the separation point, near the natural vortex shedding frequency at $F^+ = 1$. Duvigneau and Vissoneau [8] conducted an automatic optimization study of SJ parameters, controlling the SJ velocity, frequency and flow angle for a NACA 0015 airfoil flow at $Re_c = 8.96 \times 10^5$, at angles of attack between 12$^\circ$ and 24$^\circ$. The SJ’s optimal parameters were different at each angle of attack, but the angle-averaged optimized SJ parameters still improved aerodynamic performance. A maximum lift increase of 34% and stall delay from 19$^\circ$ to 22$^\circ$ was achieved when the SJ was operated at optimal parameters. Chapin and Benard [1] performed a URANS study to control a stalled flow around NACA 0012 airfoil at $Re = 10^6$. A SJ within the separation region improved lift coefficient by 84%, and delayed stall from 18$^\circ$ to 25$^\circ$. The influence of different actuator parameters on the effectiveness and robustness of control was investigated. Contrary to the majority of published research, the authors concluded that the optimal location of the actuator was not in the neighborhood of the separation point, but further downstream within the separated region, assuming the flow is dominated by two-dimensional unsteady flow phenomena. Torres [9] experimentally investigated control of flow over a NACA 65(2)-415 airfoil at chord Reynolds numbers between $Re_c = 150,000$ and
Rec = 450,000, by placing the SJ near the trailing edge on the pressure side of the airfoil. The pressure side SJ operated like a Gurney flap, and turned the trailing edge shear layer. SJ control increased lift by 7% and lift-to-drag ratio by 15%. Lardeau and Leschziner [5] performed an LES study of a three dimensional turbulent boundary layer separating from a rounded ramp in a duct forced by a pair of spanwise-periodic round SJs, actuated upstream of the nominal separation line. SJ control reduced the length of the separated region and thickness of the reverse-flow layer. Flow control was the most effective when SJs were actuated against the free stream, close to the separation zone and in close spanwise locations.

Low-Reynolds number flows over an airfoil, such as the one over a NACA 65(1)-412 airfoil considered in this paper, separates from the airfoil suction side, and induces vortex-dominated wakes. In such vortex-dominated flows, coherent structures play a critical role in transport physics that impact design considerations. Identifying and understanding the behavior of coherent structures and important transport barriers are essential to a successful control of flow separation.

Lagrangian Coherent Structure (LCS) theory has proven to be a useful method for identifying coherent structures, transport barriers and time-dependent structures [10] of vortex dominated flows. Haller [11, 12] and Haller and Yuan [13] introduced Finite-Time Lyapunov Exponent (FTLE) theory that relates LCS with regions of locally maximal repulsion or attraction, and demonstrated that FTLE fields quantitatively measure the repulsion rate in forward-time and the attraction rate in backward-time. Shadden et al. [14] identified LCS using Finite-Time Lyapunov Exponent (FTLE) fields, by defining LCS as ridges of FTLE fields. Shadden et al. demonstrated that FTLE fields do not always exhibit behavior that is Lagrangian, and therefore the generalization that FTLE fields always identify LCS is inaccurate. However, the FTLE field is still a useful diagnostic tool for identifying structures that may be LCS or other important structures (shear layers) in unsteady flows [15].

In this paper, we investigate the SJ control of a flow past a NACA 65(1)-412 airfoil and the associated FTLE fields at a chord Reynolds number of Rec = 20,000. A baseline flow and SJ flow cases are solved with DNS of the Navier-Stokes equations discretized with a discontinuous-Galerkin (DG) spectral element method. The FTLE fields are computed in a finite difference algorithm in parallel to the DNS to investigate the qualitative effects of the SJ on the flow physics. The algorithm can determine two and three dimensional FTLE fields, based on the dimensions of the input velocity data. Distributions of pressure and skin friction, along with time series of lift and drag, contours of pressure and vorticity, and boundary layer velocity profiles further help analyze the effects of SJ control. To the best of our knowledge, this is the first study on the control of ridges in the FTLE field over an airfoil. Three SJ control strategies are presented in this paper. Firstly, a SJ is simulated on the airfoil suction
side, slightly downstream of the separation point and actuated normal to the separated shear layer. Secondly, a SJ near the leading edge is simulated, normal to the airfoil suction surface. Thirdly, the airfoil pressure side, trailing edge SJ control, similar to that found in Torres [9] for high Reynolds number flows, is simulated.

The next chapter presents the governing equations and models. In chapter 3, the setup of the airfoil and SJ modeling is discussed. Results and discussions are presented in chapter 4. Conclusions of the research are drawn in chapter 5.
CHAPTER 2
GOVERNING EQUATIONS AND MODEL

2.1 GOVERNING EQUATIONS

2.1.1 Dimensional Form

The two dimensional flow over NACA 65(1)-412 airfoil considered in this paper is Newtonian and compressible. The flow is governed by the Navier-Stokes equations of the form

\[ \dot{Q}^* + \nabla \cdot F^* = 0, \]  \hspace{1cm} (2.1)

where \( Q^* \) is a vector of conserved variables and \( F^* \) is the flux tensor. \( F^* \) can be decomposed into advective \((a)\) and viscous \((v)\) fluxes, as follows.

\[ \nabla \cdot F^* = F^a_{x} + G^a_{y} - (F^v_{x} + G^v_{y}) \]  \hspace{1cm} (2.2)

The conserved variables are

\[ Q^* = \begin{bmatrix} \rho^* \\ \rho^* u^* \\ \rho^* v^* \\ \rho^* e^* \end{bmatrix}. \]  \hspace{1cm} (2.3)

The total energy is the sum of internal and kinetic energies, \( \rho^* e^* = \rho^* c_v T^* + \rho^*(u^2 + v^2)/2 \). The advective fluxes are given by

\[ F^{a*} = \begin{bmatrix} \rho^* u^* \\ p^* + \rho^* u^* v^* \\ \rho^* u^* v^* \\ u^*(\rho^* e^* + p^*) \end{bmatrix}, \quad G^{a*} = \begin{bmatrix} \rho^* v^* \\ \rho^* v^* u^* \\ p^* + \rho^* v^* v^* \\ v^*(\rho^* e^* + p^*) \end{bmatrix}. \]  \hspace{1cm} (2.4)

The viscous fluxes are

\[ F^{v*} = \begin{bmatrix} \tau^*_{xx} \\ \tau^*_{yx} \\ u^* \tau^*_{xx} + v^* \tau^*_{yx} + \kappa^* T^*_{x} \end{bmatrix}, \quad G^{v*} = \begin{bmatrix} \tau^*_{xy} \\ \tau^*_{yy} \\ \tau^*_{yy} \\ u^* \tau^*_{xy} + v^* \tau^*_{yy} + \kappa^* T^*_{y} \end{bmatrix}. \]  \hspace{1cm} (2.5)
Components of shear stress in (2.5) are defined as
\[
\tau_{xx}^* = 2\mu^* \left( u_x^* - \frac{u_x^* + v_y^*}{3} \right)
\]
\[
\tau_{yy}^* = 2\mu^* \left( v_y^* - \frac{u_x^* + v_y^*}{3} \right)
\]
\[
\tau_{xy}^* = \tau_{yx}^* = \mu^* \left( v_x^* + u_y^* \right)
\]
(2.6)

Air as an ideal gas is used in the simulation of the airfoil flow. Pressure, density and temperature are related by the ideal gas law, \( p^* = \rho^* R T^* \), where \( R \) is the particular gas constant of air.

### 2.1.2 Non-Dimensional Form

The airfoil flow DNS solves a non-dimensional form of the Navier-Stokes Equations introduced in 2.1.1. The non-dimensional Navier-Stokes Equations are constructed by normalizing the variables of the dimensional Navier-Stokes Equations with reference values of length \( (L_f) \), density \( (\rho_f) \), velocity \( (U_f) \), temperature \( (T_f) \), viscosity \( (\mu_f) \), and thermal diffusivity \( (\kappa_f) \). The non-dimensional variables are provided below.

\[
x = \frac{x^*}{L_f}
\]
\[
y = \frac{y^*}{L_f}
\]
\[
u = \frac{u^*}{U_f}
\]
\[
v = \frac{v^*}{U_f}
\]
\[
\kappa = \frac{\kappa^*}{\kappa_f}
\]
\[
T = \frac{T^*}{T_f}
\]
\[
\mu = \frac{\mu^*}{\mu_f}
\]
\[
\rho = \frac{\rho^*}{\rho_f}
\]
\[
p = \frac{p^*}{\rho_f U_f^2}
\]
(2.7)

For the present study, \( L_f \) is the chord, \( U_f \) is the free stream velocity, \( \rho_f \) is the density of air as ideal gas at reference state, and the reference temperature is \( T_f = 200K \). The reference Reynolds, Mach, and Prandtl numbers are

\[
Re_f = \frac{\rho_f U_f L_f}{\mu_f} = 20000
\]
\[
Ma_f = \frac{U_f}{(\gamma RT_f)^{1/2}} = 0.3
\]
\[ (2.8) \]
\[
Pr_f = \frac{C_p \mu_f}{\kappa_f} = 0.72
\]

The non-dimensional equation of state is \( p = \rho T / \gamma Ma_f^2 \), where \( \gamma = 1.4 \), is the ratio of specific heats. Using a constant Prandtl number approximation, the viscosity \( \mu \) and the thermal diffusivity \( \kappa \) are computed by Sutherland’s law,

\[
\mu = \kappa = \frac{(1 + R_T)T^{3/2}}{T + R_T}
\]
(2.9)
where $R_T$ is the ratio of the Sutherland constant and the reference temperature: $R_T = S/T_f = 110/200$. With the outlined normalization, the non-dimensional Navier-Stokes Equations take the following form.

$$Q + F_x^a + G_y^a - \frac{1}{Re_f}(F_x^v + G_y^v) = 0$$  \hspace{1cm}(2.10)

The vector of conserved variables is

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix}.$$  \hspace{1cm}(2.11)

The advective fluxes are

$$F^a = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ \rho v \\ u(\rho e + p) \end{bmatrix}, \quad G^a = \begin{bmatrix} \rho v \\ \rho vu \\ p + \rho v^2 \\ v(\rho e + p) \end{bmatrix},$$  \hspace{1cm}(2.12)

and the viscous fluxes are

$$F^v = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{yx} \\ u\tau_{xx} + v\tau_{yx} + \frac{\kappa}{(\gamma - 1)PrM_f^2}T_x \end{bmatrix}, \quad G^v = \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ u\tau_{xy} + v\tau_{yy} + \frac{\kappa}{(\gamma - 1)PrM_f^2}T_y \end{bmatrix}.$$  \hspace{1cm}(2.13)

The non-dimensional total energy is given by $\rho e = p(\gamma - 1) + \rho(u^2 + v^2)/2$. The components of the non-dimensional stress tensor are as follow.

$$\tau_{xx} = 2\mu(u_x - (u_x + u_y)/3)$$
$$\tau_{yy} = 2\mu(v_y - (u_x + v_y)/3)$$
$$\tau_{xy} = \tau_{yx} = \mu(v_x + u_y)$$  \hspace{1cm}(2.14)

### 2.1.3 The Discontinuous-Galerkin Spectral Element Method

The Navier-Stokes Equations of Section 2.1.2 are discretized with a discontinuous-Galerkin spectral element method. The computational domain $\Omega$ is divided into a set of non-overlapping quadrilateral elements $D_k$,

$$\Omega = \bigcup_{k=1}^{K} D_k.$$  \hspace{1cm}(2.15)
As shown in Figure 2.1, an element $D_k$ of an arbitrary shape in physical space $x = (x, y)$ is mapped to a unit square in computational coordinates $\xi = (\xi, \eta)$, using a transfinite map according to the following linear blending formula

$$
\begin{align*}
x(\xi, \eta) &= (1 - \eta) \Gamma_1(\xi) + \eta \Gamma_3(\xi) + (1 - \xi) \Gamma_4(\eta) + \xi \Gamma_2(\eta) \\
&\quad - x_1(1 - \xi)(1 - \eta) - x_2\xi(1 - \eta) - x_3\xi\eta - x_4(1 - \xi)\eta.
\end{align*}
$$

(2.16)

The functions $\Gamma_i$ are parametric curves that bound the elements. The mapping is uniquely defined by the boundary polynomial representation $\Gamma_i$ and the corner coordinates $x_i$. The schematic of this mapping is provided in Figure 2.1.

Figure 2.1. Mapping of an element of an arbitrary shape from the physical space $(x, y)$ to a unit square in the computational space $(\xi, \eta)$. Source: [16]

Under this mapping, the Navier-Stokes Equations (2.10) become

$$
\tilde{Q}_t + \nabla \cdot \tilde{F} = 0,
$$

(2.17)

where

$$
\begin{align*}
\tilde{Q}_t &= JQ \\
\tilde{F}_i &= (a_j \times a_k)_i \cdot F.
\end{align*}
$$

(2.18)

The $a_i$ vectors are covariant basis vectors, as shown in Figure 2.2, defined by $a_1 = \partial x / \partial \xi$ and $a_2 = \partial x / \partial \eta$, where

$$
\begin{align*}
\frac{\partial x}{\partial \xi} &= x_1(1 - \eta) - x_2(1 - \eta) - x_3\eta + x_4\eta + \Gamma'_1(1 - \eta) + \Gamma'_3\eta + \Gamma_2 - \Gamma_4 \\
\frac{\partial x}{\partial \eta} &= x_1(1 - \xi) + x_2\xi - x_3\xi - x_4(1 - \xi) + \Gamma'_2\xi + \Gamma'_4(1 - \xi) - \Gamma_1 + \Gamma_3
\end{align*}
$$

(2.19)
The terms $\Gamma'_i$ in (2.19) are the derivatives of $\Gamma_i$ with respect to their arguments. The term $J$ in (2.18) is the Jacobian of the mapping,

$$ J = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta}. $$

(2.20)

The $N$th-order Legendre polynomials (2.21) in the computational coordinates $(\xi,\eta)$ approximates the mapped solution vector $\tilde{Q}$,

$$ \tilde{Q}_N(\xi, \eta, t) = \sum_{i=0}^{N} \sum_{j=0}^{N} L_i(\xi)L_j(\eta)\tilde{Q}_{i,j}(t) = \sum_{i=0}^{N} \sum_{j=0}^{N} \hat{Q}_{i,j}(t)\ell_i(\xi)\ell_j(\eta), $$

(2.21)

where $\ell_i(\xi)$ and $\ell_j(\eta)$ are Lagrange polynomials given by

$$ \ell_i(\xi) = \prod_{k=0}^{N} \frac{(\xi - \xi_k)}{(\xi_j - \xi_k)}. $$

(2.22)

In the Galerkin method, the solution and flux vectors are required to be orthogonal to the test function $\phi(\xi, \eta) = \ell_i(\xi)\ell_j(\eta)$. A system of ordinary differential equations

$$ \mathcal{M} \frac{\partial \tilde{Q}_N}{\partial t} + \mathcal{F}_{i,j} - \mathcal{S}_{i,j} = 0 $$

(2.23)

can be constructed after integration by parts and using the following definition of matrices.

$$ \mathcal{M}_{i,j} = \int_{D_k} \ell_i(\xi)\ell_j(\eta)d\xi $$

$$ \mathcal{F}_{i,j} = \int_{\partial D_k} \ell_i(\xi)\ell_j(\eta)dS $$

$$ \mathcal{S}_{i,j} = \int_{D_k} \nabla(\ell_i(\xi)\ell_j(\eta))d\xi $$

(2.24)
\( M \) is the mass matrix, \( S \) is the stiffness matrix, and \( F^\ast \) is the numerical flux which is the solution to an approximate Riemann problem at the coincident faces from two neighboring subdomains. The ordinary differential equations (2.23) are integrated in time with a 4th-order Runge-Kutta method.

### 2.2 Lagrangian Coherent Structures and Finite-Time Lyapunov Exponents

Following [11, 14, 17], the discussion of Finite-Time Lyapunov Exponents (FTLE) starts by considering a velocity field provided by the following ordinary differential equation and initial condition,

\[
\dot{x}(t) = v(x(t), t) \quad x_0 = x(t_0).
\]  (2.25)

\( v(x(t), t) \) is the Eulerian velocity field of the fluid, and \( x(t) \) is the trajectory of a fluid particle. The solution to (2.25) is

\[
x(t) = x_0 + \int_{t_0}^{t} v(x(\tau), \tau) d\tau.
\]  (2.26)

(2.26) can be re-written in terms of the initial fluid particle trajectory \( x_0 \), the initial time \( t_0 \), and the time interval \( T \equiv t - t_0 \), to yield

\[
x(x_0, t_0; T) = x_0 + \int_{t_0}^{t_0+T} v(x(\tau), \tau) d\tau.
\]  (2.27)

The flow map, \( \phi \), maps material points from their initial location \( x_0 \) and initial time \( t_0 \) to their location at a time \( t = t_0 + T \),

\[
\phi(x_0, t_0; T) \equiv x(x_0, t_0; T)
\]  (2.28)

The flow map is a function of the space-time \( (x_0, t_0) \), also known as the extended phase space, determined over the time interval \( T \). The extended phase space for an \( n \)-dimensional geometry is denoted as \( X \subset \{\mathbb{R}^n, t\} \). The particle trajectory equations and the flow map are related by defining the flow map to be the set of all values of \( x \) constrained by (2.27) within \( X \):

\[
\phi(x_0, t_0 : T) := \{ x \in \mathbb{R}^n : x = x_0 + \int_{t_0}^{t_0+T} v(x(\tau), \tau) d\tau, \forall \{x_0, t_0\} \in X \}
\]  (2.29)

The Taylor expansion of the flow map defined in (2.28) is used to define the linearized flow map, \( \bar{\phi} \). The linearized flow map describes the mapping of perturbations in \( x_0 \) over the period \( T \). The Taylor expansion of (2.28) is

\[
\phi(x_0, t_0; T) \approx \phi(y_0, t_0; T) + \frac{\partial \phi(y_0, t_0; T)}{\partial x_0} (x_0 - y_0) + O((x_0 - y_0)^2).
\]  (2.30)
The perturbation \( \delta x \) is defined in the limit \( y_0 \to x_0 \),

\[
\delta x_0 \equiv \lim_{y_0 \to x_0} |x_0 - y_0|.
\] (2.31)

Similarly, a differential change in the flow map is defined

\[
\delta \phi \equiv \lim_{y_0 \to x_0} |\phi(x_0, t_0; T) - \phi(y_0, t_0; T)|.
\] (2.32)

Then, the linearized flow map is defined by the first-order Taylor expansion,

\[
\bar{\phi}(x_0, t_0; T) : \delta \bar{\phi} = \frac{\partial \bar{\phi}(x_0, t_0; T)}{\partial x_0} \delta x_0 = \nabla_{x_0} \bar{\phi} \delta x_0
\] (2.33)

From this point, it is assumed \( \phi(x_0, t_0; T) = \bar{\phi}(x_0, t_0; T) \). Because \( \nabla_{x_0} \phi \) is a linear operator on \( \delta x_0 \), the operator norm can be used to find the maximal amount that the operator will stretch an initial line element of length \( |\delta x_0| \),

\[
\max |\delta \phi| = \|\nabla_{x_0} \phi\| |\delta x_0|.
\] (2.34)

With the right Cauchy-Green strain tensor \( C \) and its maximal eigenvalue \( \lambda_{max}(C) \), the operator norm is

\[
\|\nabla_{x_0} \phi\| = \sqrt{\lambda_{max}(\nabla_{x_0} \phi^* \nabla_{x_0} \phi)} = \sqrt{\lambda_{max}(C)}
\] (2.35)

Under the assumption that \( \max |\delta \phi| \) diverges exponentially in time, the rate of this divergence is determined from

\[
\max |\delta \phi| = |\delta x_0| e^{\sigma T},
\] (2.36)

where \( \sigma \) is the FTLE. A more precise definition is constructed by rearranging terms in (2.36).

\[
\sigma(x_0, t_0; T) = \frac{1}{|T|} \ln \|\nabla_{x_0} \phi\| = \frac{1}{|T|} \ln \sqrt{\lambda_{max}(C)}
\] (2.37)
Equation (2.37) clearly shows that the FTLE is a measure of the maximum stretching rate in the fluid. The use of the absolute value of the integration time $T$ in (2.37) permits use of both positive and negative integration times [14]. A positive integration time reveals repelling LCS normal to which fluid particles are stretched, and a negative integration time computes attracting LCS along which fluid particles are attracted. Nelson’s [15] depiction of the repelling and attracting LCSs to be identified by FTLE fields is provided in Figure 2.3.
CHAPTER 3
NUMERICAL MODEL

3.1 AIRFOIL SETUP

The flow over a NACA 65(1)-412 airfoil presented in this paper is at a chord Reynolds number $Re_c = 20,000$, Mach number $Ma = 0.3$ and angle of attack $\alpha = 4^\circ$. The computational domain is enclosed by a semi-circular inlet surface at a radial distance of $5c$ from the trailing edge, horizontal surfaces $5c$ away from the trailing edge, and a vertical surface $15c$ downstream of the trailing edge, as shown in Figure 3.1 (a). An inflow condition is specified at the inlet surface, and pressure outflow conditions are specified at all other surfaces. The airfoil surface is an adiabatic wall where a no-slip condition is specified.

![Figure 3.1. The (a) computational domain and (b) mesh for the airfoil DNS. Source: [15]](image-url)
The airfoil geometry is mapped using a cubic spline constructed from published coordinates of the NACA 65(1)-412 airfoil. The DG elements at the airfoil boundary are approximated with curved-sided elements. The mesh consists of 2256 elements, where the ratio of the largest and the smallest element area is $A_{\text{max}}/A_{\text{min}} = 9.68 \times 10^3$. The mesh is shown in Figure 3.1 (b). The grid is $p-$ refined to a polynomial order of $N = 12$. As the polynomial order is increased, properties of the airfoil flow, such as skin friction profile and peak vortex shedding frequency, coalesce to demonstrate the grid-independence of the flow physics and verify the quality of the mesh, as shown in Figure 3.2.

![Figure 3.2. Convergence of the frequency spectrum (a) and average airfoil suction surface skin friction profile (b) with grid $p-$ refinement. Source: [15]](image)

The aerodynamic forces on the airfoil are computed using Gauss quadrature to ensure a high-order accuracy of the force calculation [16]. The axial force $A$ along the airfoil chord and the normal force $N$ normal to the airfoil chord are determined by

\[
A = \sum_{k=1}^{K_b} \int_S (\tau_{xx} n_x + \tau_{yx} n_y - p n_x) dS
\]

\[
= \sum_{k=1}^{K_b} \sum_{i}^{N} (\tau_{xx} n_x + \tau_{yx} n_y - p n_x) w_i dS_i,
\]

\[
N = \sum_{k=1}^{K_b} \int_S (\tau_{xy} n_x + \tau_{yy} n_y - p n_y) dS
\]

\[
= \sum_{k=1}^{K_b} \sum_{i}^{N} (\tau_{xy} n_x + \tau_{yy} n_y - p n_y) w_i dS_i.
\]

(3.1)
where \( \tau_{ij} \) is the shear stress tensor given in 2.14, \( p \) is the pressure, \( \mathbf{n} \) is the outward normal, \( w_i \) is the quadrature weight, and \( dS_i \) is the differential arc-length of an element given by

\[
d S_i = \sqrt{\left(\frac{\partial x}{\partial \xi^k} \left(\xi_i^\tau\right)\right)^2 + \left(\frac{\partial y}{\partial \xi^k} \left(\xi_i^\tau\right)\right)^2}
\]  

(3.2)

The lift and drag coefficients \( C_L \) and \( C_D \) are obtained from the forces by

\[
C_L = \frac{N \cos \alpha - A \sin \alpha}{1/2}
\]

\[
C_D = \frac{N \sin \alpha - A \cos \alpha}{1/2},
\]

(3.3)

where \( \alpha \) is the angle of attack. The DNS records the lift and drag coefficients as a time series at every time step.

The pressure and skin friction on the airfoil are computed by projecting the pressure and shear stress tensor to the boundary using Lagrange interpolation. Pressure is extracted directly, and the skin friction is determined from the stress at the boundary by

\[
C_f = \frac{1}{Re_f} \{ -(\tau_{xx} n_y + \tau_{yx}) n_x + (\tau_{xy} n_x + \tau_{yy} n_y) n_x \}
\]

(3.4)

To study the effect of different SJ configurations on the airfoil flow, the baseline flow was initially developed for 10 time units. This allows the flow to go through transients and flow features to develop and settle at a quasi-steady state. Then, the baseline and SJ controlled flows were simulated for 20 time units from \( t = 10 \) for analysis.

### 3.2 Synthetic Jet Setup

Suzuki et al. [18] introduced effects of periodic mass injection by artificially forcing the right-hand side of the Navier-Stokes equations. The SJ in this study is modeled similarly, where source terms are introduced to the momentum and energy conservation equations to reflect the behavior of the SJ. Pasquetti and Peres [19] note that due to the small size of the SJ, the mass fluctuation from the SJ is very small. They reason that because the net mass flux associated with the SJ is zero, and taking into account the mass fluctuation with a source term is difficult when high order methods are involved, it is reasonable to introduce no source term to the continuity equation. Thus, the mass source term in this paper is set to zero.

The impact force of fluid from the SJ onto the free stream flow can be formulated as

\[
\mathbf{F} = \dot{m} \mathbf{u}_{jet},
\]

(3.5)

where the mass injected into the flow by the SJ is \( \dot{m} = \rho |\mathbf{u}_{jet}| A_{jet} \). \( A_{jet} \) is the width of the SJ actuator on the airfoil surface, and \( \mathbf{u}_{jet} \) is the SJ velocity vector. Therefore, the impact force
from the SJ onto the freestream is

\[
F = \rho |u_{jet}| A_{jet} u_{jet}.
\]

(3.6)

Pasquetti and Peres [19] suggest transforming the impact force of the SJ as the volume integral of the momentum source term \( m_{jet} \),

\[
\int_{D_{jet}} m_{jet} d\Omega = F,
\]

(3.7)

where \( D_{jet} \) is the computational region within which the SJ forcing is simulated. Following Suzuki et al. [18], the source terms are Gaussian-distributed in an elliptical region of the \( xy \)-plane in the vicinity of the SJ. The SJ coordinate system \((x_1, y_1)\) is defined such that the major axis, \( y_1 \) in this study, is along the jet direction. In terms of the SJ coordinates \( x_1 \) and \( y_1 \), the Gaussian-distributed momentum source term has the form

\[
m_{jet}(x_1, y_1) = \frac{F}{2\pi \sigma_{x_1} \sigma_{y_1}} \exp\left[-\frac{(x_1 - x_s)^2}{2\sigma_{x_1}^2} - \frac{(y_1 - y_s)^2}{2\sigma_{y_1}^2}\right]
\]

(3.8)

where \( \sigma_{x_1}, \sigma_{y_1} \) are respectively lengths of the semi-minor and semi-major axes, and \( x_s \) and \( y_s \) are the coordinates for the center of the elliptical forcing region. Suzuki et al. [18] used half of the inlet diffuser height, and Dandois et al. [2] used the ramp height as the reference length scales by which to scale the size of the forcing region. These reference lengths are scales normal to the free stream flow direction, so the maximum thickness of the NACA 65(1)-412 airfoil, \( 0.12c \), is used as the reference length in this study. Scalings of \( \sigma_{x_1}/0.12c = 0.01 \) and \( \sigma_{y_1}/0.12c = 0.08 \) [18] are set for the forcing region. The elliptical forcing region for the pressure side, trailing edge SJ is shown in Figure 3.3.

Figure 3.3. Elliptical forcing region of the pressure side, trailing edge SJ
The SJ frequency is determined from scale of the wake vortices, whose physics we want to control. Assuming that the diameter of the shed vortices are comparable to the vertical projection of the airfoil at an angle of attack of \(4^\circ\), the diameter of the vortices is

\[ d = c \sin(\alpha) = \sin(4^\circ). \]  
(3.9)

These vortices have a timescale of

\[ T = d/U_\infty = \sin(4^\circ). \]  
(3.10)

The inverse of this timescale is the vortex shedding frequency,

\[ F^+ = 1/T = 14.3. \]  
(3.11)

This number is similar to the frequency reported by Sandham and Touber [20]. This \(F^+\) is introduced into the DNS as a circular frequency, \(\omega = 2\pi F^+\). The lowest free stream velocity of \(U_\infty = 17.88\) m/s and the peak SJ velocity of \(|u_{\text{jet}}| = 30\) m/s from Torres [9] are used to construct the SJ velocity modeling of \(|u_{\text{jet}}(t)| = 1.68 \sin(\omega t)\) in the DNS.

The source terms are introduced to the Navier-Stokes equations (2.10), to yield

\[ \partial_t Q + F_x^u + G_y^u - \frac{1}{Re_f} (F_x^v + G_y^v) = S, \]  
(3.12)

where \(S\) is the vector of the source terms. The source term of the energy equation is obtained as the sum of the products of velocity and momentum source components. The source term vector has the form

\[ S = \begin{bmatrix} 0 \\ m_{\text{jet}}(x_1, y_1)_x \\ m_{\text{jet}}(x_1, y_1)_y \\ u m_{\text{jet}}(x_1, y_1)_x + v m_{\text{jet}}(x_1, y_1)_y \end{bmatrix}, \]  
(3.13)

where \(m_{\text{jet}}(x_1, y_1)_x\) and \(m_{\text{jet}}(x_1, y_1)_y\) are the \(x\) and \(y\) components of the momentum source term \(m_{\text{jet}}(x_1, y_1)\). These source terms model the three SJ cases introduced in Chapter 1.
CHAPTER 4
RESULTS AND DISCUSSIONS

4.1 BASELINE

The forward and backward-time FTLE fields as shown in Figure 4.1, identify the LCS and other material lines of importance of the airfoil flow. As introduced in Chapter 2.2, the forward-time FTLE identifies repelling material lines, also known as stable manifolds, normal to which fluid particles are stretched. Near the leading edge, the forward-time FTLE delineates the stable manifold that extends to the stagnation point, which illustrates the leading edge separation line where the flow separates normal to it to travel either over or under the airfoil (Fig. 4.1 (a)). This provides an insight into the flow structure akin to the angle of attack of the incoming flow. The backward-time FTLE identifies attracting material lines, also known as unstable manifolds, tangent to which fluid particles stretch. Lipinski et al. [21] explain that an unstable manifold attached to a wall identifies where fluid is drawn from the near wall region and ejected along the material line. Therefore, the flow separation line should correspond with an unstable manifold identified by the backward-time FTLE. The reverse is also true, where a stable manifold attached to a wall, identified by the forward-time FTLE, corresponds to flow reattachment. They also observe that fluid particles move onto or very near the unstable manifolds. Hence, the backward-time FTLE is also a good visual tool for identifying vortex structures in the wake, as shown in Figure 4.1 (b).

The baseline airfoil flow, without SJ control, separates from the airfoil suction surface around $x = 0.5c$, as shown in Figure 4.1. A recirculation zone forms between the separated shear layer and the airfoil surface from the flow separation point to the trailing edge. Vortices are alternatingly shed from the airfoil suction and pressure surfaces and form a time-periodic, asymmetric von Karman vortex street in the wake. Vortices that are shed from the pressure side are stronger than those shed from the suction side, evidenced by their clearer spiral structure (Fig. 4.1), greater FTLE magnitudes, and larger vorticity values (Figure 4.2 (c)). The strength difference of the vortices results in a lower pressure and density in the pressure side vortex cores compared to those of suction side vortex cores, as depicted in Figure 4.2 (a), (b).
Figure 4.1. Forward-time (a) and backward-time (b) FTLE fields of the baseline airfoil flow

The repelling LCS show that the fluid above the shear layer, denoted (1) in Figure 4.3, is entrained into the upper surface vortex as it develops and sheds in the recirculation region, denoted (2) in Figure 4.3. Region (1) travels downstream with the outer flow and entrains more fluid into the suction surface vortex as it develops over time. Attracting LCS (Fig. 4.4) are aligned with the braid regions that connect the Karman vortices. The near-vertical attracting and repelling LCS on the airfoil suction surface, regions denoted (3) in Figures 4.3 and 4.4 are time-independent and form transport barriers that prevent any fluid contained in the recirculation region from advecting downstream [15]. The main trailing edge vortex on the suction surface is denoted (4) in Figure 4.4 and is nearly constant in size over its shedding cycle.
Figure 4.2. Contours of (a) pressure, (b) density, and (c) vorticity of the baseline airfoil flow
Figure 4.3. The repelling LCS identified by the forward-time FTLE field of the baseline flow in the recirculation region and the near wake
Figure 4.4. The attracting LCS identified by backward-time FTLE field of the baseline flow in the recirculation region and the near wake

The RMS velocity field quantifies the fluctuations of fluid velocity from the time-averaged velocity, as shown in Figure 4.5, over a time period. The highest RMS velocity values are observed in the wake region, where the flow is dominated by the downstream-advecting, time-periodic vortices that have high rotational energy and hence high changes in the velocity field. The RMS velocity around the airfoil, upstream of the wake region, is low, and hence demonstrates that the flow field around the airfoil is quasi-steady. Some velocity fluctuations are observed in the part of the recirculation region close to the trailing edge, where the suction surface vortex sheds.

Figure 4.5. The RMS velocity field of the baseline flow computed from $t = 15$ to $t = 30$
4.2 Synthetic Jet Control

For a thorough analysis of SJ control effects, the forward-time and backward-time FTLE fields, time series of lift and drag, distributions of pressure and skin friction, contours of pressure and vorticity, root-mean-squared (RMS) velocity field, and boundary layer $u$-profiles are compared between the baseline flow and each of the SJ control cases.

4.2.1 Suction Side Synthetic Jet: Normal To Separated Flow

A SJ is simulated on the airfoil suction surface at $x/c = 0.6$, downstream of the separation point, normal to the separated shear layer. The SJ penetrates and perturbs the shear layer, and the high-frequency perturbations are carried downstream by the outer flow, as seen in the region denoted (1) in Figure 4.6 (b). The SJ destabilizes the recirculation zone and increases vorticity development (Fig. 4.7), and causes high fluctuations of the wake direction (Fig. 4.6). This development of recirculation region vorticity grows the main trailing edge vortex, and induces additional vortices upstream. The growth of recirculation region vorticity gives rise to as many as 3 vortices instantaneously (V1, V2, and V3 in Fig. 4.8 (a), (b), (c)).

Figure 4.6. Forward-time (a) and backward-time (b) FTLE fields of the airfoil flow with suction surface SJ actuated normal to the shear layer at $x/c = 0.6$
Figure 4.7. Unsteady vorticity development of the airfoil flow with suction surface SJ actuated normal to the shear layer at $x/c = 0.6$.

The forward-time FTLE field shows the upstream propagation of SJ perturbations, labeled (1) in Figure 4.9. The elongated forward-time FTLE attached to the airfoil surface ((2) in Fig. 4.9) show that increase of vorticity caused by the SJ induces greater flow reattachment than was observed for the baseline flow. This in turn reduces the pressure of the recirculation region (Fig. 4.10 (a), (b)), and increases the pressure difference between airfoil surfaces to increase lift. At maximum lift situations, the length of reattached flow is nearly doubled (Figs. 4.3 (a), 4.9(b)) from baseline flow.
Figure 4.8. Backward-time FTLE in the near-wake region near peak lift situation (a), (b), (c), and near minimum lift situation (d) of the airfoil flow with suction surface SJ at $x/c = 0.6$.

The instantaneous pressure difference between the two airfoil surfaces is the greatest at situations where vorticity in the recirculation region is the greatest (Fig. 4.10 (c)). This corresponds to a maximum lift situation that occurs with a period of $T \approx 2.5$, as seen in Figure 4.11 (a). As the 3 suction side vortices shed, 2 of them shed consecutively. This abruptly reduces the vorticity and increases pressure in the recirculation region (Figs. 4.7 (f), 4.10 (d)). This significantly drops lift (Fig. 4.11 (a)), immediately following the aforementioned lift peaks. The growth of the recirculation region vorticity grows the vortices, and increases blockage by the shear layer (Fig. 4.8 (a), (b), (c)), and increases the pressure drag of the airfoil. Conversely, as the developed vortices shed, the recirculation region shrinks and deflects the shear layer downwards, reducing blockage and hence the pressure drag. Therefore, instances of significant peaks and dips of lift and drag are simultaneous, and occur is periodic with $T \approx 2.5$ (Fig. 4.11).
Figure 4.9. Forward-time FTLE of the near-wake region of the airfoil flow with suction surface SJ actuated normal to the shear layer at $x/c = 0.6$

Figure 4.10. Pressure development of the airfoil flow with suction surface SJ actuated normal to the shear layer at $x/c = 0.6$
Figure 4.11. The time series of (a) lift and (b) drag for the airfoil flow with suction surface SJ actuated normal to the shear layer at x/c = 0.6

On a time-averaged comparison with the baseline flow, the suction surface pressure is lower near the trailing edge (Fig. 4.13 (a)), due to longer flow reattachment caused by increased vorticity of the recirculation region. The pressure on the rest of the airfoil suction surface is higher, and on the pressure surface is lower. The time-averaged skin friction profiles (Fig. 4.13) shows that there is a marked increase of skin friction at the SJ actuator and high fluctuations near it due to upstream and downstream propagation of high-frequency SJ perturbations. The pressure surface skin friction is nearly identical to that of the baseline flow. The collapse of the airfoil pressure profile is the result of a reduction of angle of attack, explained in Figure 4.12. As depicted in the time-averaged leading edge separation line (Fig. 4.14), the SJ reduces the time-averaged angle of attack for the SJ flow by 4.83%.

Figure 4.12. Mechanism by which angle of attack is reduced for the airfoil flow with suction surface SJ actuated normal to the shear layer at x/c = 0.6
Figure 4.13. Time-averaged profiles of (a) pressure, (b) skin friction on airfoil suction surface, and (c) skin friction on airfoil pressure side for the airfoil flow with suction surface SJ actuated normal to the shear layer at $x/c = 0.6$.

The time-averaged $u$-velocity profiles demonstrate that the SJ perturbations propagated upstream thicken the boundary layer prior to separation (Fig. 4.15 (a), (b)), and that the higher time-averaged vorticity generated by SJ control deflects the shear layer upwards and thickens the recirculation region (Fig. 4.15 (c), (d)). The increased time-averaged vorticity is also portrayed by greater local minimum and maximum velocity magnitudes observed within the recirculation region (Fig. 4.15 (c), (d)). As tabulated in Table 4.1, the lift and drag-increasing vorticity growth of the recirculation region and the lift-decreasing reduced angle of attack work against each other to reduce the total time-averaged lift by 2.98%, and increase the total time-averaged drag by 5.21% compared to
the baseline flow. The SJ reduces pressure lift and skin friction lift by 2.97% and 5.35%, and increases the pressure drag and skin fraction drag by 6.52% and 3.00% respectively.

Figure 4.14. The time-averaged forward-time FTLE of the (a) baseline flow and (b) the airfoil flow with suction surface SJ actuated normal to the shear layer at $x/c = 0.6$

The RMS velocity field shown in Figure 4.16 demonstrates a marked increase of velocity fluctuations within the recirculation region (Fig. 4.5), and quantifies the unsteadiness of the recirculation zone vortex physics. The high fluctuation of the wake direction caused by the SJ makes the high-RMS region in the wake thicker compared to that of the baseline. The RMS wake region shows that over time, the SJ causes a greater upward deflection than downward of the wake compared to the baseline flow’s wake.
Figure 4.15. Time-averaged $u$ profiles at (a) $x/c = 0.6$, (b) $x/c = 0.7$, (c) $x/c = 0.8$, and (d) $x/c = 0.9$ for the airfoil flow with suction surface SJ actuated normal to the shear layer at $x/c = 0.6$
4.2.2 Suction Side Synthetic Jet: Near The Leading Edge

A SJ on the airfoil suction surface at $x/c = 0.1$ is simulated normal to the airfoil surface. This SJ actuation aims to control flow separation by modifying the flow before it transitions to turbulence and subsequently separates. The perturbations to the boundary layer from SJ forcing are observed both upstream and downstream of the actuator prior to flow separation (Fig. 4.17). The SJ penetrates the boundary layer at the actuator and is advected by the outer flow, so that downstream of flow separation, the SJ effects are outside of, but attached to, the separated shear layer. The flow within the recirculation region is not significantly affected by the SJ, and hence its FTLE fields are similar to those of the baseline flow.
Figure 4.17. Forward-time (a) and backward-time (b) FTLE fields of the airfoil flow with suction surface SJ actuated normal to the airfoil surface at $x/c = 0.1$.

Figure 4.18. Contours of (a) pressure and (b) vorticity for airfoil flow with suction surface SJ actuated normal to the airfoil surface at $x/c = 0.1$.

The pressure contour (Fig. 4.18 (a)) demonstrates the similitude between the baseline flow and the current SJ flow case. As seen in the vorticity contour of Figure 4.18 (b), the SJ produces a street of small vortices with alternating vorticity. The increased vorticity on the airfoil suction surface from these vortices turns the wake downwards compared to that of the baseline flow. The SJ actuation reduces the angle of attack, as detailed in Figure 4.19, in a similar mechanism that caused the angle of attack reduction in the SJ case of the previous section. The time-averaged leading edge separation line (Fig. 4.21) depicts a $4.26\%$ reduction of the angle of attack. This reduction is to a smaller degree than observed for the SJ flow case.
of the previous section, because the vicinity of the SJ actuator creates a smaller cumulative pressure reduction than the recirculation region does (Figs. 4.13 (a), 4.20 (a)). The decrease of angle of attack causes a slight increase of suction side pressure upstream of flow separation. The time-averaged pressure distribution on the rest of the airfoil is similar to that of the baseline flow, except for the slight collapse due to the angle of attack reduction.

![Diagram](https://via.placeholder.com/150)

**Figure 4.19.** Mechanism by which angle of attack is reduced for the airfoil flow with suction surface SJ actuated normal to the airfoil surface at $x/c = 0.1$

The periodic fluid injection and withdrawal at the SJ actuator results in an increased time-averaged local skin friction at the actuator (Fig. 4.20 (b)). The perturbations introduced by the SJ to the boundary layer effectively thickens the boundary layer and reduces the velocity gradient, resulting in the reduction of the skin friction coefficient. Downstream of the separation point, the suction surface skin friction is nearly identical to that of the baseline flow as the small vortices are dissipated (Fig. 4.18 (b)). The skin friction of the airfoil pressure side displays no significant changes (Fig. 4.20 (c)).
The vortices generated by the SJ introduce a larger time-averaged $u$-velocity fluctuation at the edge of the boundary layer (Fig. 4.22 (a), (b)). The small vortices generated by the SJ are attached to the separated shear layer, and increases the thickness of the shear layer, as shown in Fig. 4.22 (c), (d). The increase in suction side vorticity also turns the von Karman vortex street of the wake downwards compared to that of the baseline flow. The SJ reduces the time-averaged lift by 1.80% and the time-averaged drag by 1.84%. The pressure lift is reduced by 1.80%, and the skin friction lift is increased by 0.56%, while the pressure drag is reduced by 2.78%, and skin friction drag is reduced by 0.25%. The current SJ case improves the drag and has a smaller lift loss compared to the SJ case discussed in the previous section.
In contrast to the SJ within the recirculation zone introduced in the previous section, this SJ is actuated where the flow is laminar. In this regime, the boundary layer is thinner as the flow has not transitioned to turbulence, and is attached to the airfoil surface. Therefore, the SJ is able to penetrate into the outer potential flow more than the SJ in the previous section is able to. This results in a pronounced effect of the SJ that is clearly seen as it is propagated upstream (Fig. 4.23) and downstream, outside the separated shear layer (Fig. 4.17). The SJ effects do not influence the vortex physics of the recirculation region, and as a result, the flow is periodic with one frequency like the baseline case, as shown in the lift and drag time series in Figure 4.24.

Figure 4.21. The time-averaged forward-time FTLE of the (a) baseline flow and (b) the airfoil flow with suction surface SJ actuated normal to the airfoil surface at $x/c = 0.1$
Figure 4.22. Time-averaged $u$-profiles of the airfoil suction side at (a) $x/c = 0.2$, (b) $x/c = 0.3$, (c) $x/c = 0.8$, $x/c = 0.9$ for the airfoil flow with suction surface SJ actuated normal to the airfoil surface at $x/c = 0.1$

The RMS velocity field, shown in Figure 4.25, displays resemblance to that of the baseline flow. The key differences between the two are velocity fluctuations from the small vortex street generated by the SJ, and the downward deflection of the wake RMS region. Low RMS velocity around the airfoil confirm the steadiness of the flow. RMS velocity due to the small vortices diminish as the flow moves downstream toward the separation point, beyond which the fluctuations due to the small vortex street are not noticeable.
Figure 4.23. Upstream propagation of SJ effects, depicted by an instantaneous forward-time FTLE near the leading edge

Figure 4.24. The time series of (a) lift and (b) drag for the airfoil flow with suction surface SJ actuated normal to the airfoil surface at $x/c = 0.1$
Figure 4.25. RMS velocity field of the airfoil flow with suction surface SJ actuated normal to the airfoil surface at $x/c = 0.1$ computed from $t = 15$ to $t = 30$

4.2.3 Pressure Side Trailing Edge Synthetic Jet

The pressure side SJ is actuated normal to the airfoil surface at $x/c = 0.87$. The SJ penetrates the boundary layer, and produces perturbations that advect downstream to add to the pressure side vortices. The forward and backward-time FTLE fields show that SJ effects are more pronounced downstream of the SJ than upstream, as seen in Figure 4.26. The wake is time-periodic and unidirectional, much like that of the baseline flow. The effects of the SJ on the flow structures are localized to the airfoil pressure surface and the vortices that shed from it.

Figure 4.26. The forward-time (a) and backward-time (b) FTLE fields of the airfoil flow with pressure surface SJ actuated normal to the airfoil surface at $x/c = 0.87$
The time-averaged profiles of pressure and skin friction (Fig. 4.27) further demonstrate that the effects of the SJ are localized to the vicinity of the actuator. Except in the vicinity of the SJ, no changes to the pressure and skin friction distributions are observed compared to baseline flow. At the SJ, the outwardly normal SJ decreases the time-averaged pressure (Fig. 4.27 (a)). The high velocity gradient induced by the SJ actuation causes a skin friction spike immediately upstream of itself, and the thickening of the downstream boundary layer by the addition of SJ perturbations decreases the skin friction downstream of it (Fig. 4.27 (b)).

Figure 4.27. Time-averaged profiles of (a) pressure, (b) skin friction on airfoil suction surface, and (c) skin friction on airfoil pressure surface for the airfoil flow with pressure surface SJ actuated normal to the airfoil surface at $x/c = 0.87$. 
Because the SJ effects are localized to the trailing edge region of the pressure surface, there is no distinguishable difference in the time-averaged velocity profile of the baseline flow and the SJ flow case at different suction surface locations (Fig. 4.28). The effects of SJ on the airfoil lift and drag are only seen as fluctuations at the SJ frequency within the main vortex shedding cycle, and no changes to the overall trend are observed for the lift and drag, as seen in Figure 4.29.

Figure 4.28. Time-averaged $u$-profiles of the airfoil suction side at (a) $x/c = 0.2$, (b) $x/c = 0.3$, (c) $x/c = 0.8$, $x/c = 0.9$, for the airfoil flow with pressure surface SJ actuated normal to the airfoil surface at $x/c = 0.87$. 
Figure 4.29. The time series of (a) lift and (b) drag for the airfoil flow with pressure surface SJ actuated normal to the airfoil surface at $x/c = 0.87$.

The RMS velocity field (Fig. 4.30) shows a nearly identical field as the baseline flow, except for a slightly thicker region of RMS near the SJ actuator. This confirms the locality of SJ effects and largely undisturbed airfoil flow structure.

Figure 4.30. RMS velocity field of the airfoil flow with pressure surface SJ actuated normal to the airfoil surface at $x/c = 0.87$ computed from $t = 15$ to $t = 30$.

As summarized in Table 4.1, the pressure side SJ reduces the total time-averaged lift by 0.47%, and increases the time-averaged drag by 0.20%. The pressure lift and skin friction lift are reduced by 0.47% and 1.00% respectively, while the pressure drag and skin friction drag respectively increased 0.08% and 0.40%. These changes are quite smaller than those observed for the two previously discussed SJ flow cases, and demonstrates that the local effects of the pressure side SJ don’t significantly change the airfoil performance. The thickening of the airfoil pressure surface boundary layer downstream of the actuator increases
the pressure drag by 0.65%. The negligible changes to the airfoil performance are in agreement with the findings of Torres [9] for high-Reynolds number flows, where a very similar SJ setup resulted in negligible changes to the lift and drag at a low angle of attack of $\alpha = 5^\circ$. The authors observed performance improvements as the angle of attack increased, so it is fair to conclude that the current simulation is insufficient to disregard the efficacy of the pressure side, trailing edge SJ for control of low-Reynolds number airfoil flows.

### 4.3 Summary of Results

Table 4.1 summarizes the components of aerodynamic forces acting on the airfoil, averaged between $t = 20$ and $t = 30$. For brevity, abbreviations are used in the table. Sside SJ refers to the suction side SJ at $x/c = 0.6$, actuated normal to the separated shear layer. Sside LE SJ denotes the SJ actuated at $x/c$, normal to the airfoil surface, on the airfoil suction surface. Pside TE SJ corresponds to the pressure side SJ at $x/c = 0.87$, actuated normal to the airfoil surface.

<table>
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<th></th>
<th>Baseline</th>
<th>Sside SJ</th>
<th>Sside LE SJ</th>
<th>Pside TE SJ</th>
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<tr>
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<td>0.018202</td>
<td>0.018320</td>
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<td>Total Drag</td>
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<td>0.051603</td>
<td>0.048148</td>
<td>0.049146</td>
</tr>
</tbody>
</table>
CHAPTER 5
CONCLUSION

A control of the flow over a NACA 65-(1)412 airfoil using synthetic jets was numerically studied. A suction side synthetic jet normal to the separated shear layer, a suction side synthetic jet normal to the airfoil surface near the leading edge, and a pressure side synthetic jet near the trailing edge were simulated respectively. Different synthetic jet flow cases were modeled by introducing source terms to the conservation equations. A baseline flow without synthetic jets, and synthetic jet flow cases were solved with direct numerical simulation of Navier Stokes equations discretized by a high-order Discontinuous Galerkin spectral element method. A finite difference algorithm identified Lagrangian Coherent Structures by computing Finite-Time Lyapunov Exponent fields in parallel and helped identify the effect of the synthetic jet on flow structures. Contours of pressure and vorticity, time series of lift and drag, time-averaged distributions of pressure and skin friction on the airfoil surface, and time-averaged velocity profiles, were additionally analyzed to study the effect of synthetic jet control.

The baseline airfoil flow separates from the suction surface approximately mid-chord, and the wake is a unidirectional, time-periodic, asymmetric von Karman vortex street shed alternatingly from the airfoil pressure and suction surfaces. A main trailing edge vortex forms within the recirculation region, the size of which is nearly invariant over time.

The suction side synthetic jet actuated normal to the separated shear layer destabilizes the vortex physics within the recirculation region. The recirculation region vorticity grows over time, and the main trailing vortex grows in size and induces additional vortices upstream. This increased vorticity decreases abruptly when the developed suction side vortices shed, and the increase in vorticity continues over time. This growth of recirculation region vorticity promotes flow reattachment and increases lift, while the growth of vortex size increases the pressure drag as the vortices deflect the shear layer further from the airfoil surface and increases blockage. The increased blockage results in an increased pressure drag. The decreased pressure of the recirculation region from growing vorticity also pulls the potential outer flow over the airfoil suction surface toward itself and reduces the angle of attack, collapsing the pressure profile of the airfoil and decreasing lift.

The leading edge, suction side synthetic jet creates a street of small vortices that stay outside, but remain attached to, the boundary layer and the separated shear layer. The additional vorticity on top of the shear layer deflects the shear layer closer to the airfoil.
surface. The low pressure zone in the vicinity of the synthetic jet actuator pulls the outer potential flow and reduces the angle of attack, slightly collapsing the profile and reducing lift. The vortices atop the shear layer decreases the pressure drag.

The pressure side, trailing edge synthetic jet minimally affected the flow and produced no global effects to the pressure and skin friction distribution at the current angle of attack. Therefore, no significant changes to the airfoil lift and drag were observed. As this is a synthetic jet that is the furthest downstream, and therefore at the highest Reynolds number of the three synthetic jet cases considered, a higher synthetic jet momentum is likely needed to produce any significant changes to the airfoil performance. It is also possible that this synthetic jet could create a low pressure zone and pull the outer potential flow below the airfoil pressure surface toward it, and increase the effective angle of attack to create a pressure lift increase. To further study the effectiveness of the pressure side synthetic jet, cases with different angles of attack as well as synthetic jet velocities should be analyzed.

Future studies of the low-Reynolds synthetic jet control should focus on optimizing parameters of the synthetic jet such that desirable effects of control observed in this study are isolated. Desirable synthetic jet control effects for enhancement of lift are increased vorticity within the recirculation region and increase of the effective angle of attack. The desired effect of synthetic jet control for drag reduction is the downward deflection of the shear layer. To achieve all of these desired effects, some combinations of synthetic jet schemes are proposed. Two suction side synthetic jets, one near the leading edge and another within the shear layer, may be set up such that the vorticity of the recirculation is increased and potentially stabilized to provide improved lift while the vortex street generated by the leading edge synthetic jet suppresses the upward deflection of the shear layer to reduce drag. A pressure side synthetic jet may be actuated in conjunction with a suction side synthetic jet in the recirculation region to take the benefit of the increased recirculation zone vorticity and prevent/increase the angle of attack to maximize lift enhancement. A successful synthetic jet control requires optimization of many parameters that also depend on the flow conditions, and an intensive analysis is crucial to achieve successful control of low Reynolds number airfoil flows.
BIBLIOGRAPHY


