VECTOR FIELD GENERATOR FOR A DIRECT MAPPING OF THE FIRST ORDER POINCARÉ SPHERE

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Melanie Anne Pierce
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The Undersigned Faculty Committee Approves the
Thesis of Melanie Anne Pierce:

Vector Field Generator for a Direct Mapping of the First Order Poincaré Sphere

[Signatures and names]

Jeffrey Davis, Chair
Department of Physics

Fridolin Weber
Department of Physics

Ricardo Carretero
Department of Mathematics and Statistics

5/12/12

Approval Date
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DEDICATION

To Aaron, for magical pancakes and teaching me that it is okay to take breaks.
ABSTRACT OF THE THESIS

Vector Field Generator for a Direct Mapping of the First Order Poincaré Sphere

by

Melanie Anne Pierce

Master of Science in Physics

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This thesis presents an optical system able to generate all polarization states on the zero order Poincaré sphere. An important characteristic of the zero order sphere is its spatial uniformity. This means that the polarization of the beam is uniform. This characterization can be proven using polarizers. Any change in the polarization of the beam will be consistent for all points in the beam. This is not necessarily true for all types of polarization states. There are new polarization states that are spatially variant in which the polarization is no longer uniform. The assumption that the polarization at one point in the beam is the same at all points is no longer valid. These are defined as higher order polarization states which have their own Poincaré spheres that are similar to the zero order sphere but are spatially variant. The higher order polarization states are the focus of this thesis. Maxwell’s equations are shown and the solution for light is derived. From this, Jones vectors are used to describe the polarization and how they relate to the Poincaré sphere. Jones matrices are applied to the incoming polarization state to reflect the changes a waveplate causes to the system, and how to create a rotator to rotate the axis of polarization. The matrices describe an optical system consisting of a variable waveplate and a rotator created from 2 quarter waveplates and an additional variable waveplate that able to change the latitude and longitude of a polarization state on the Poincaré sphere. The system is able to achieve any coordinate on the surface of the sphere. The system is applied to the zero order Poincaré sphere and the positive and negative first order Poincaré sphere. Experimental results are presented and agree with theory.
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CHAPTER 1

INTRODUCTION

The Poincaré sphere is a familiar concept for visualizing all the different possible elliptically polarized states. For the zero order Poincaré sphere the beams are characteristically uniform. The polarization is consistently uniform throughout the beam. This is not necessarily true for all types of polarization states. There are fairly new polarization states that are spatially variant in which the polarization is no longer spatially uniform. The assumption that the polarization at one point in the beam is the same at all points is no longer valid, which allows for unique behavior in the beam. The spatially variant polarization states are defined as cylindrical vector beam polarization states which have their own Poincaré spheres that are similar to the zero order sphere but spatially variant. The Poincaré spheres that represent these cylindrical vector beams are known as higher order Poincaré spheres. The polarization states on the first order Poincaré sphere are the primary focus of this thesis and will be analyzed in comparison to the zero order case and the differences between them will become apparent.

The spatially variant polarization states on the first order Poincaré sphere have a strong interest in the field of optics. The spatial variant beams possess unique behavior such as producing very small focal spots when focused down, giving them a strong application in other fields such as spectroscopy,\(^1\) particle acceleration,\(^2\) microscopy,\(^3\) and optical trapping.\(^4\) Outside of these fields, the polarization states have been found naturally in crystal optics,\(^5\) Mie scattering,\(^6\) and the cosmic background radiation.\(^7\)

In this thesis, I show a simple optical system that is capable of generating any polarization state represented on the zero order Poincaré sphere. That same system is applied to the first order Poincaré sphere to cause the same manipulation of the beam. The system
consists of a variable waveplate to change the phase of the output, and a rotator constructed of quarter waveplates and an additional variable waveplate which rotates the angle of the direction of the polarization.

In the first chapter of this thesis I explain in detail how polarized light is created starting with Maxwell’s equations. I then explore the vector representation of polarization states, and the vectors on surface of the Poincaré sphere. The polarization states on the sphere are characterized by latitude and longitude coordinates like those found on the earth. The longitude lines stretch along the sphere from the north to south poles. The latitude lines run parallel to the equator of the sphere perpendicular to the longitude, therefore any location on the sphere can be characterized by a latitude and longitude. After explaining the Poincaré sphere, I describe how Jones matrices can be used to show how to change from one state to another. The Jones matrices can produce any polarization on the sphere by traveling along the longitude to either poles, and by traveling along the latitude or the equator. The Jones matrices represent changes to the system caused by optical systems such as a variable waveplate and a rotator. Using devices that produce such changes to the system, I can change the polarization state from one state to another. I first utilize this knowledge with linear plane wave polarizations that are mapped on the zero order Poincaré sphere.

With the zero order sphere, I prove that the system created with a waveplate and a rotator can move effectively around the Poincaré sphere and produce any desired elliptically polarized beam. The same optical system can theoretically be applied to the first order Poincaré spheres, with the addition of a radial polarization converter.

After proving that the system works for the zero order sphere, I describe the difference between the zero order Poincaré sphere and the first order Poincaré sphere. An important characteristic of the zero order sphere is its spatial uniformity. This means that the polarization of the beam is uniform. This characteristic can be proven using polarizers to analyze the beam. Any change in the polarization caused by the analyzers will be consistent for all points in the beam. This is not necessarily true for all types of polarization states. There are cylindrical vector beam polarizations that are spatially variant and the polarization is no longer uniform.
The final part of this thesis is applying the system from the zero order onto the first order states produced using the radial polarization converter. The results will be analyzed, and conclusions about the success of the system will be made. The appendices describe how to identify when the variable waveplates that was used in this thesis are at the proper phase difference as well as another means of describing the polarization states and unexpected results found with the higher order states. Next we review the description of the common zero order polarization states.
CHAPTER 2

MAXWELL'S EQUATIONS

2.1 Deriving the Wave Equation

Light waves are electromagnetic waves and therefore abide by the same rules as other electromagnetic waves and can be explained by Maxwell equations. These are written as follows

\[ \nabla \cdot D = \rho , \]
\[ \nabla \cdot B = \rho , \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} , \]
\[ \nabla \times H = J + \frac{\partial B}{\partial t} . \]

In this familiar set of equations, \( E \) is the electric field, \( B \) is the magnetic displacement, \( H \) is the magnetic field, \( D \) is the electric displacement and \( \rho \) and \( J \) are the electric charge density and current density respectively. We can define the three vectors by

\[ J = \sigma E , \]
\[ D = \varepsilon_0 E + P = \varepsilon E , \]
\[ B = \mu_0 (H + M) = \mu H . \]

In this system \( P \) is the polarizability, \( \varepsilon \) is the permittivity, \( M \) is the magnetization and \( \mu \) is the permeability. In free space the variables are written as \( \varepsilon_0 \) and \( \mu_0 \). For simplicity I will assume that \( \rho \) and \( J \) are zero which is true in most cases.
Using the equations above, the wave equation can be derived using the vector identity
\[ \nabla \times (\nabla \times A) = \nabla \cdot (\nabla \cdot A) - \nabla^2 A. \] (2.8)
For the electric field the output is
\[ \nabla^2 E = \mu \varepsilon \frac{\partial^2 B}{\partial t^2}. \] (2.9)
Using the same method for the magnetic field, the output is of a similar form:
\[ \nabla^2 B = \mu \varepsilon \frac{\partial^2 E}{\partial t^2}. \] (2.10)

The simplest case for the solutions to the wave equation is to assume plane wave solutions. More complicated solutions exist for optical fibers and waveguides, but they are not relevant to this experiment. For a plane wave, each component, generally denoted by \( \phi_i \), of both the electric and magnetic field are governed by the wave equation. The wave equation for each of the vector components of both the magnetic and electric field is separately written for each component as:
\[ \nabla^2 \phi_i = \mu \varepsilon \frac{\partial^2 \phi_i}{\partial t^2}. \] (2.11)
In most cases, the propagation direction is assumed to be in the \( z \) direction such that the solutions to the differential equation above are of the form \( \phi_i = f(\nu t \pm z) \) where \( \nu \) is the velocity of the waves. By this assumption, a solution is only possible if the velocity is:
\[ \nu = \frac{\omega}{k} = \frac{1}{\sqrt{\mu \varepsilon}}, \] (2.12)
where \( \omega \) and \( k \) are the angular frequency and the wave vector respectively. In a vacuum this value is equal to the speed of light \( c \). If the wave is traveling through a medium, then
\[ \nu = \frac{c}{n} = \frac{\sqrt{\varepsilon_0 \mu_0}}{\sqrt{\varepsilon \mu}} c, \] (2.13)
where \( n \) is the index of refraction and depends on \( \varepsilon \), \( \mu \) and \( \varepsilon_0 \) and \( \mu_0 \).
The vector component $\phi_i$ from Equation (2.11) can be rewritten without the speed by substituting Equation (2.12) so the solution will be $\phi_i = f(\alpha t - kz)$, which is assumed to be a sinusoidal function.

After rewriting in exponential complex form, the $E$ and $B$ fields are described by:

$$E(x, y, z, t) = (E_x \hat{i} + E_y \hat{j} + E_z \hat{k}) e^{i(\alpha t - kz)}, \quad (2.14)$$

$$B(x, y, z, t) = (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) e^{i(\alpha t - kz)}. \quad (2.15)$$

From the Maxwell equations it is known that $\nabla \cdot E = 0$ can be expressed from the above equations as $-i\hat{k} \cdot E = 0$. This implies that the electric field is perpendicular to the $\hat{k}$ vector, which means it has no component along the $z$ axis which is the direction of propagation. The same relationship exists for the $B$ field. Equations (2.14) and (2.15) can be simplified to

$$E(x, y, z, t) = (E_x e^{i\phi_i} \hat{i} + E_y e^{i\phi_j} \hat{j}) e^{i(\alpha t - kz)}, \quad (2.16)$$

$$B(x, y, z, t) = (B_x e^{i\phi_i} \hat{i} + B_y e^{i\phi_j} \hat{j}) e^{i(\alpha t - kz)}. \quad (2.17)$$

Here $e^{i\phi}$ is the phase component of the wave, and $e^{i(\alpha t - kz)}$ is the propagation behavior of the wave.

### 2.2 Light propagation in a Medium

In the previous section, we defined the electric field and magnetic field as they propagate through a medium where the velocity is given by $v = \sqrt{\frac{\epsilon}{\mu}}$. Both $\mu$ and $\epsilon$ depend on the medium through the magnetization and polarization in Equations (2.6) and (2.7). Normally the magnetization can be ignored.

The polarization field $P$ of the medium defines how the molecules of an object move when interacting with an external electric field. In general, there is a tensor relationship between $P$ and $E$ as

$$P_x = E_0 (\chi_{11} E_x + \chi_{12} E_y + \chi_{13} E_z), \quad (2.18)$$

$$P_y = E_0 (\chi_{21} E_x + \chi_{22} E_y + \chi_{23} E_z), \quad (2.19)$$
\[ P_z = \varepsilon_0 (\chi_{31} E_x + \chi_{32} E_y + \chi_{33} E_z). \]  

(2.20)

To simplify the system, the polarization tensor is diagonalized such that only the diagonal components are nonzero in a new coordinate system. The above equations reduce to

\[ P_1 = \varepsilon_0 (\chi_{11} E_1), \]  

(2.21)

\[ P_2 = \varepsilon_0 (\chi_{22} E_2), \]  

(2.22)

\[ P_3 = \varepsilon_0 (\chi_{33} E_3), \]  

(2.23)

and these can now be substituted into Equation (2.6) to get the components for \( D \),

\[ D_1 = \varepsilon_1 E_1, \]  

(2.24)

\[ D_2 = \varepsilon_2 E_2, \]  

(2.25)

\[ D_3 = \varepsilon_3 E_3. \]  

(2.26)

The indices of refraction for each axis are defined as

\[ n_1 = \sqrt{\frac{\varepsilon_{11}}{\varepsilon_0}}, \]  

(2.27)

\[ n_2 = \sqrt{\frac{\varepsilon_{22}}{\varepsilon_0}}, \]  

(2.28)

\[ n_3 = \sqrt{\frac{\varepsilon_{33}}{\varepsilon_0}}. \]  

(2.29)

The relationships above imply that the velocity of light \( v = c/n \) depends on the direction of propagation of the beam. These relationships can be described by the index ellipsoid seen below in Figure 2.1 where the three indices of refraction in the ellipsoid are not necessarily the same by must satisfy Equation (2.30) below.
Figure 2.1: Index ellipsoid describing ray propagation dependence on axis.\(^9\)

The equation to describe the index ellipsoid is written as,

\[
\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1.
\] (2.30)

Many materials are isotropic where all the indices are the same and the index ellipsoid is just a sphere. In uniaxial crystals, two of the axes are identical and one axis is not the same resulting in \(n_1 = n_2 = n_o \neq n_e\). The two equal axes are called the ordinary axes because they resemble an isotropic material, but the third is called the extraordinary axis and is either larger or smaller than the other two. If the wave’s direction of propagation is parallel to the extraordinary axis, then the material will behave isotropically and only interact with the ordinary axis. If it is traveling along one of the ordinary axes, then the polarization will behave differently depending on which axis it is perpendicular to. In a more general sense, if the propagation direction is at an angle \(\theta\) with respect to the major axis of the index ellipsoid, then the size of the major axis will vary as

\[
\frac{1}{n_e^2(\theta)} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}.
\] (2.31)

If the incident beam is propagating along \(n_o\) in the \(z\) direction, the \(E\) field has components in two orthogonal directions traveling with different velocities. The solution to the wave equation is given by,
\[ E(z,t) = (E_x e^{i\varphi_0} \hat{i} + E_y e^{i\varphi_0} \hat{j}) e^{i(\alpha z - \omega t)}. \] (2.32)

In Equation (2.32), \( \varphi_{0x} \) and \( \varphi_{0y} \) are the phases of each component. Both the size and the initial phase of the \( E \) field components could be changed. In the next chapter I will show a more efficient way to denote the electric field.
CHAPTER 3

JONES VECTORS AND THE POINCARÉ SPHERE

3.1 Jones Vectors

Jones vectors are a way to represent the vector form of light, and is the starting foundation of the geometric representation known as the Poincaré Sphere. To understand how to describe light with Jones vectors we can rewrite the Equation (2.32) from the previous chapter into vector form;

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{bmatrix}. $$

(3.1)

In Equation (3.1) $\phi_x$ and $\phi_y$ are the relative phase of each component. In the case of linearly polarized light where $\phi_x = \phi_y$, we can reduce the vector down to the $x$ axis and $y$ axis polarizations,

$$E_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{(3.2)}$$

$$E_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad \text{(3.3)}$$

The light is polarized in either the $x$ or $y$ direction so that as it travels along the $z$ direction the polarization will be oriented either $x$ and $y$ as shown in Figures 3.1 and 3.2.
The polarization state of the beam can be oriented anywhere along the $x$, $y$ plane to produce a plane wave polarization at a given angle so that

$$E = \begin{bmatrix} E_{0x} \cos \theta \\ E_{0y} \sin \theta \end{bmatrix},$$

(3.4)

where $\theta$ is any arbitrary angle from the $x$ axis.

One of the more common states between vertical and horizontal polarizations is the case when the light is polarized at $\pi/4$ radians. The normalized vector is in this case is

$$E_{\pm \pi/4} = \frac{1}{\sqrt{2}} \begin{bmatrix} \pm 1 \\ 1 \end{bmatrix}.$$  

(3.5)
The ± sign in Equation (3.5) denotes the direction of the light in either the $+\pi/4$ direction or the $-\pi/4$ direction.

The polarization state in Equation (3.1) can become more complex with the addition of a relative phase difference. The relative phase difference is caused when the $x$ and $y$ components are out of phase with each other. This can produce a variety of polarization states with varying radians of phase difference between the $x$ and $y$ components.

When the light is perfectly circularly polarized, the $x$ and $y$ components are completely out of phase with one another. The phases can be related to each other by

$$
\varphi_y = \varphi_x \pm \frac{\pi}{2}.
$$

(3.6)

The ± sign determines the direction of the circularly polarized light. From this left circularly polarized light can be expressed as

$$
E_L = \begin{bmatrix}
E_{0x} e^{i\varphi_x} \\
E_{0y} e^{i(\varphi_y - \pi/2)}
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 \\
-i
\end{bmatrix}.
$$

(3.7)

Likewise, right circularly polarized light is written as

$$
E_R = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 \\
i
\end{bmatrix}.
$$

(3.8)

In a similar fashion as the linear polarizations, an infinite number of possible states exist in between right and left circularly polarized light that depend on the relative phase difference between $x$ and $y$ components. These states are generally called elliptically polarized light and can be described by:

$$
E_E = \frac{1}{\sqrt{2}} \begin{bmatrix}
E_x e^{i\varphi} \\
E_y e^{i\varphi}
\end{bmatrix},
$$

(3.9)

where $E_x \neq E_y$ and/or $\varphi \neq \pi/2$. The axes of the ellipse are determined by its linear polarization component. The major/minor axes of the ellipse can be in the $x$ and $y$ direction or rotated like that of linear polarization states. Examples of the difference between linear, circular and elliptical can be seen in Figure 3.3 below.
3.2 Zero Order Poincaré Sphere

The Poincaré sphere is a different method to represent the state of polarization. The Poincaré sphere was created in 1892 by Jules Henri Poincaré. He showed that the state of polarization of a beam of light can be described as a point on a unit sphere.

Just like any sphere, on the Poincaré sphere there is an equator and two poles which indicate important polarization states as well as any point on the surface of the sphere is a unique polarization state. Lines that are parallel to the equator are the latitude lines, and the lines that run perpendicular to the equator are the longitude lines. Starting on the left side of the sphere in Figure 3.4 and traveling along the longitude upward, the polarization state changes from vertical to elliptical to right circular. Similarly, traveling downward from the left side, the polarization state changes from vertical to elliptical to left circular. All along this longitude line the axis of the elliptically polarized light is in the vertical direction starting with a very thin ellipse and expanding outward until it reaches the poles and is purely circular. For any elliptically polarized state the axis can be rotated by traveling along the latitude. The elliptical polarization axis will go from vertical to positive $\pi/4$ radians to horizontal to negative $\pi/4$ radians and back to vertical.

On the sphere the ellipticity can be changed by traveling along a longitude, and the axis direction can be changed by traveling along a latitude line. The states described above are coordinates on the most basic Poincaré sphere, which is known as the zeroth order in Figure 3.4.
The Poincaré sphere is created through a connection between the optical angular momentum of the beam and the resulting polarization. On the Poincaré sphere in Figure 3.4 the only angular momentum of interest is the spin angular momentum of ±ℏ per photon which corresponds to the circular polarizations at the poles of the Poincaré sphere. The two points at the poles are the eigenstates of the spin angular momentum, and is a basis for all plane wave coordinates on the zeroth order sphere as well as for higher orders, which will be shown later. The order of the sphere is determined by the orbital angular momentum known as ℓℏ; (ℓ = ±1, ±2, ±3…±n). The case for ℓ = 0 produces the states in Figure 3.4. There is no orbital angular momentum, therefore we have uniform polarization throughout the beam and create the linear polarization states described above with Jones vectors.

Any state of polarization can be described not only by the Jones vector but an azimuthal angle φ and the ellipticity θ. The coordinates on the sphere can be described by latitude and longitude as (2θ, 2φ), due to the π/4 polarization states occurring at π/2 on the sphere. The Jones vector for the general form of polarized light can be expressed by these coordinates, starting with the inclusion of the latitude,
\[ E = \begin{bmatrix} E_0 \cos(\phi)e^{i\varphi_x} \\ E_0 \sin(\phi)e^{i\varphi_y} \end{bmatrix}. \]  

(3.10)

The exponential term can be expanded into sine and cosine terms utilizing the property that

\[ e^{\pm i\varphi} = \cos \varphi \pm i \sin \varphi, \]  

(3.11)

and expanded into Equation (3.10) so that,

\[ E(\phi) = \begin{bmatrix} \cos \phi \cos \theta - i \sin \phi \sin \theta \\ \sin \phi \cos \theta + i \cos \phi \sin \theta \end{bmatrix}, \]  

(3.12)

where the relative phase difference \( \varphi \) has been converted to \( \theta \) to represent the overall ellipticity where \( \varphi = 2\theta \).

Each vector that corresponds to a point on the zero order Poincaré sphere can be produced at certain values of \( \phi \) and \( \theta \). The points on the equator occur when \( \theta = 0 \) and \( \phi \) is set to some angle so that linearly polarized light is oriented at the angle and there is no ellipticity or relative phase difference between the \( x \) and \( y \) components. One can travel along the equator of the sphere and from vertical to horizontal polarization by changing \( \phi \) or rotating the system. The introduction of a nonzero \( \theta \) is what causes ellipticity in the beam and produces circular polarization when \( \theta = \pi \), and includes the complex component of the vector. The polarization states of the zero order sphere can be seen below in Figure 3.5 along with their latitude and longitude coordinates. Figure 3.5 shows that changing the latitude rotates around parallel to the equator and changing the longitude changes the ellipticity and moves between the poles.
An important characteristic of the zero order sphere is its spatial uniformity. The entire beam is polarized in a single direction. The states of interest in this thesis are states that no longer have the spatial uniformity of the zero order states. The spatially variant beams are part of the first order Poincaré sphere. These states will be discussed more in Chapter 6 and the spatial variant properties will be observed.

I have shown the relationship between the polarization and the ellipticity and azimuthal angle, but not how to actually manipulate the angles to achieve any desired polarization state. This can be done with Jones matrices that correspond to physical optical devices used to experimentally manipulate the beam.
CHAPTER 4

JONES MATRICES AND WAVEPLATES

4.1 Jones Matrices

The Jones vectors described in Section 3.1 describe the polarization direction of the beam either when it enters an experimental setup, or when it leaves the setup. The zero order Poincaré sphere showed how the polarization state can be changed by moving around the equator or upward to the poles.

To manipulate the beam, Jones matrices are used in conjunction with the Jones vectors to produce a new polarization. Each optical element is represented by a 2x2 matrix that multiplies the incident electric field to produce a new $E$ field. An optical system can be explained with a series of matrices applied in order of interaction with the beam;

$$E = [B][A]E_0.$$  \hspace{1cm} (4.1)

In Equation (4.1) the input beam $E_0$ travels through an optical device described by matrix $[A]$ and then through a different device described by matrix $[B]$ and the output polarization is determined. All of our systems can be explained using three basic matrices.

4.2 Linear Polarizer

The matrix that corresponds to a linear polarizer oriented to pass light in the $x$-direction is given by

$$P_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \hspace{1cm} (4.2)$$

A linear polarizer only allows light to pass through that is in the same orientation as the polarizer, and blocks the polarization that is perpendicular to the polarizer’s orientation.
4.3 Waveplate oriented in $x$ and $y$ Directions

A waveplate consists of birefringent material which has different indices of refraction for the different directions. Recall the discussion from Chapter 2 on the index ellipsoid and materials with different indices of refraction. For a waveplate, two of the indices are the same, and a third is different. The ordinary and extraordinary axes are sometimes referred to as the principle axes. The simplest scenario for finding the Jones matrix for a wave plate is when the principle axes are the $x$ and $y$ axes. What makes a waveplate different from a linear polarizer is that the different indices of refraction cause a relative phase difference in the $x$ and $y$ axis. A phase difference can produce circularly polarized light from a linearly polarized input. In general a waveplate is described by

$$W_0 = \begin{bmatrix} e^{-\frac{i \omega_2 nd}{\lambda}} & 0 \\ 0 & e^{-\frac{i \omega_1 nd}{\lambda}} \end{bmatrix},$$ (4.3)

where $\lambda$ is the wavelength of $E$, $d$ is the thickness of the waveplate, and $n_o$ and $n_e$ are the indices of the ordinary and extraordinary axis as described in Chapter 2. In general the terms in front of the matrix are dropped because it is only the intensity being examined. This reduces the matrix of a waveplate to

$$W_0 = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i \phi} \end{bmatrix}. $$ (4.4)

The phase difference $\phi$ is described by

$$\phi = \frac{2\pi}{\lambda} d(n_e - n_o).$$ (4.5)

The Equation (4.4) can be re-written for a quarter wave plate, $\phi = \pi/2$ as

$$W_{\pi/2} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}.$$ (4.6)

A similar matrix can be produced for a half waveplate for $\phi = \pi$ as
$$W_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$  \hfill (4.7)

### 4.4 Rotation Matrix

A rotation matrix does what the name describes, and rotates the incoming beam while applying the properties of whatever device is being rotated. A rotation matrix is written as

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}. \hfill (4.8)$$

The output of a rotation on a matrix in the $x, y$ plane is the result of the following 3 matrices:

$$W' = R(-\theta)W_0R(\theta). \hfill (4.9)$$

$W_0$ represents the matrix of the wave plate (or any other device rotated on an axis), $R(\theta)$ and $R(-\theta)$ are the rotation and rotation inverse matrices, and $W'$ is the resulting matrix for a device rotated by the angle $\theta$. $R(\theta)$ rotates the system by $\theta$ and then $R(-\theta)$ rotates the system back after the device inside is accounted for. This system allows for axes of the polarization state to rotate to any desired angle.
CHAPTER 5

EXPERIMENTAL SETUP

5.1 Rotator and Phase Compensator

To produce any polarization state on the zero order Poincaré sphere I can use a
waveplate and a rotator to achieve the polarization states. To prepare the beam before it is
manipulated by these two it is focused through a spatial filter then expanded and collimated
and linearly polarized in the y direction as in Figure 5.1. The variable waveplate is meant to
change the phase, or the ellipticity $\theta$ and the rotator rotates the linear direction, or the
azimuthal angle $\phi$ and changes the orientation of the beam.

![Figure 5.1: Optical system for zero order Poincaré sphere.](image)

If the input polarization state is vertically polarized, then the variable waveplate will
need to be oriented at $\pm \pi/4$ to produce the phase change between the $x$ and $y$ components. A
rotator is represented by a rotation matrix, but is built with three waveplates. For the purpose
of this experiment, it is made by placing a variable waveplate at zero radians between two
crossed quarter wave plates at $\pi/4$. The two variable waveplates have their own phase
measurements that they can produce denoted by $\varphi_1$ and $\varphi_2$. Here $\varphi_1$ changes the phase and $\varphi_2$
rotates the orientation. The final setup after the beam is linearly polarized is below in Figure 5.2.

![Diagram of setup for zero order coordinates](image)

**Figure 5.2: Diagram of setup for zero order coordinates.** A HeNe laser is used as the source.

For this experiment, I want to utilize the matrices in Chapter 4 to be able to reach any desired latitude and longitude on the sphere and to confirm that the experimental setup in Figures 5.1 and 5.2 will achieve the desired results. Recall the diagram of the sphere from Chapter 3 below in Figure 5.3.
Figure 5.3: Zero order Poincaré sphere with latitude and longitude coordinates.¹¹

The latitude is caused by changes to $\phi$ or the axis of polarization and the longitude is caused by changes to $\theta$ or the ellipticity of the polarization. To travel along the longitude I can use a waveplate to produce a relative phase difference between the $x$ and $y$ components of $E$. I am starting with vertically polarized light, so the waveplate needs to be rotated by $\pi/4$. The results of a rotated waveplate can be found using the Jones matrices from the Chapter 4 for a waveplate and the rotation and rotation inverse Jones matrices to produce

\[ R \left( \frac{\pi}{4} \right) W(\phi_1) R \left( \frac{\pi}{4} \right) = \begin{bmatrix} \frac{i\phi_1}{2} & \frac{-i\phi_1}{2} \\ \frac{e^{-\theta} + e^{\theta}}{2} & \frac{e^{-\theta} - e^{\theta}}{2} \\ \frac{i\phi_1}{2} & \frac{-i\phi_1}{2} \\ \frac{e^{-\theta} - e^{\theta}}{2} & \frac{e^{-\theta} + e^{\theta}}{2} \end{bmatrix}. \]  

Equation (5.1) is for a generic waveplate with the phase difference of $\phi_1$ which corresponds to the variable waveplate in the experimental setup. Applying this transformation to the input vector polarized in the $x$ direction as in Equation (3.2) produces the vector of light in terms of the phase difference produced from the waveplate:
This vector will provide any point along the longitude starting at vertically polarized light. For $\phi_1 = \pi/2$ this vector will be circularly polarized, and any point in between will produce elliptically polarized light along the vertical axis. The longitude on the sphere is expressed by $\phi_1 = 2\theta$.

To travel on the latitude requires a tunable rotation matrix. One can be created using two waveplates that are crossed at $-\pi/4$ and $+\pi/4$ radians, with a variable waveplate in between them as seen in Figure 5.4. The variable waveplate causes small adjustments in the phase and can change the relative phase difference by increments smaller than a typical waveplate.

![Figure 5.4: Rotator made for variable rotation angles.](image)

Figure 5.4 can be written as

$$R \left( \frac{\phi_2}{2} \right) = \left[ W_{-\frac{\pi}{4}} \left( \frac{\pi}{2} \right) W_{\frac{\pi}{4}} \left( \frac{\pi}{2} \right) \right].$$

The value in the parentheses of the outside waveplates is the phase change produced by the waveplate and the subscript is the rotation of the waveplates, therefore the waveplates produce a $\pi/2$ phase shift, but are rotated by $\pi/4$. The rotation in the waveplates can be expanded out using the matrices from Chapter 4 so that,
$$R\left(\frac{\varphi_2}{2}\right) = R\left(\frac{\pi}{4}\right)W\left(\frac{\pi}{2}\right)R\left(-\frac{\pi}{4}\right)W\left(\frac{\varphi_2}{2}\right)R\left(-\frac{\pi}{4}\right)W\left(\frac{\pi}{2}\right)R\left(\frac{\pi}{4}\right),$$

(5.4)

where the rotated waveplates on either end will use Equation (5.1) to describe their system. Equation (5.4) reduces to,

$$R\left(\frac{\varphi_2}{2}\right) = \begin{bmatrix}
\cos \frac{\varphi_2}{2} & \sin \frac{\varphi_2}{2} \\
-\sin \frac{\varphi_2}{2} & \cos \frac{\varphi_2}{2}
\end{bmatrix}. \tag{5.5}
$$

Equation (5.5) is just a rotation matrix from Equation (4.8) in terms of $\varphi_2$, which is the phase of the waveplate in the middle of the system. This confirms that the rotator system will rotate the orientation of the polarization. The latitude on the sphere can be expressed as $\varphi_2 = 2\phi$ just as the longitude was expressed for the phase of the variable waveplate.

By using a variable retarder to change the phase of the input and a rotator to change the angle of plane polarization, it is possible to map all polarizations from $2\pi$ radians of plane waves up to both circular polarizations and any points in between such as varying levels of elliptical polarizations as explained in Chapter 3. This system will be invaluable when utilized with the concept of the Poincaré sphere and mapping polarizations onto the sphere. This system is simple enough to be created with common optical tools but is robust enough to be applied to more complicated polarization states. It has now been shown mathematically that this would work to map the Poincaré sphere, but it still needs to be experimentally proven as well before moving on to more complex polarization states.
CHAPTER 6

ZERO ORDER POINCARÉ SPHERE

6.1 Results

To start this experiment, I needed to verify that the rotator and variable retarder worked on the zeroth order Poincaré sphere as a proof of concept. Before the beam was manipulated by the variable waveplate and the rotator it was focused through a spatial filter, expanded, collimated and vertically polarized as described previously. After undergoing the transformations the beam was captured on a Sony CCD camera as described in the previous chapter. This allowed for clear images of the beam to show the polarization fully. Recall the final setup after linear polarization from Chapter 5 repeated in Figure 6.1. The phase shifts $\varphi_1$ and $\varphi_2$ are produced by the variable waveplates and are produced using a micrometer on the waveplates. The phase shifts $\varphi_1$ and $\varphi_2$ correspond to the ellipticity and the orientation of the polarization as discussed previously. The phase shifts are measured by a micrometer on each tunable phase shifter. The adjustment of the micrometer to find the corresponding phase shifts are described more in Appendix B.
I tested the variable wave retarder (waveplate) and the rotator using two different methods. The first was recording images of the beam through different analyzers to visualize the output polarization. The second was to graph the intensity of the beam as I changed the polarization of the analyzer to get a quantitative confirmation that the rotator and the variable wave retarder change the polarization as predicted. I recorded the phase measurements for the phase shifter and the rotation angle for the rotator for each polarization state.

After recording the measurements I took images at each polarization state and used a linear polarizer to analyze linearly polarized light and a linear polarizer with a quarter wave plate at \( \pi/4 \) radians to analyze circularly polarized light. Rotating the quarter wave plate to the opposite \( \pi/4 \) radians produced the opposite polarization analyzer. There were also plastic circular polarization detectors by Aflash Photonics for a quick check of the proper circular polarization state. The images were taken using the linear polarizer and quarter wave plate for stability in the image quality.

Cases 1-4 show results when the value of the phase retarder \( \phi_1 \) in Figure 6.1 is changed and we move along the longitude above and below the equator as shown in Figure 6.2.
Figure 6.2: States created by changing $\varphi_1$ and producing a phase change in the polarization state.

The arrows in the figures below for each case indicate the direction of the analyzer used such as vertical analyzer, or right circular, etc.

**6.1.1 Case 1: $\varphi_1 = 0$ Produces Vertical Polarization State**

The first case is the input beam that is the vertical polarization state as shown in Figure 6.2. When the input to the system is vertically polarized light, the vertical analyzer (a) should have the brightest intensity because all of the light is polarized in the vertical direction and so none is blocked by the analyzer.

The most important image is the horizontal analyzer that is expected to block all the vertically polarized light, and in Figure 6.3 (c) shows no transmitted light. The output is not as dark because the camera is detecting any ambient light at 632nm, which is normally insignificant in comparison to the laser. The $\pi/4$ radians analyzers and the circular analyzers (b) and (d) are half the intensity of the vertical analyzer (a). This is due to the $\pm \pi/4$ analyzers having equal $x$ and $y$ polarization components. Therefore half of the beam intensity is blocked by the horizontal component. The circular polarizations (e) and (f) are half the intensity as well because linearly polarized light is an equal combination of left circular and right circularly polarized light. The left circular analyzer only allows the left circular component through, which is half of the polarization. The same can be said for the right
analyzer. At this point there has been no manipulation of the beam, and the output is the same as the input.

![Images](image-url)

Figure 6.3: Vertically polarized beam through different analyzer polarizations depicted by arrows.

6.1.2 Case 2: \( \phi_1 = \pi/2 \) Produces Right Circular Polarization State

Case 2 in Figure 6.2 is the result of \( \phi_1 = \pi/2 \). This indicates that the variable waveplate is behaving like a quarter waveplate. This phase shift should have produced circularly polarized light, which would indicate being on a pole on the Poincaré sphere as seen in Figure 6.2.

Results are shown in Figure 6.4 and the beam is blocked when the left circular analyzer is present (e) because the light is right circularly polarized and is completely blocked by the left circular analyzer. The right circular analyzer (f) should produce the beam with the highest intensity, and the linear states (a)-(d) should be equal but dimmer intensities due to the nature that linear polarized light is an equal combination of right and left circular polarizations. There is right circularly polarized light therefore only half of the state is being let through the analyzer.

![Images](image-url)

Figure 6.4: Right Circularly polarized beam through different analyzer polarizations depicted by arrows.
The state has now moved upward to the right circular pole as seen in Figure 6.2. By increasing the phase shift even more, the polarization state will travel downward away from the pole in Case 2 and approach linearly polarized light again.

6.1.3 Case 3: $\varphi_1 = \pi$ Produces Horizontal Polarization State

Case 3 in Figure 6.2 shows an increased phase difference so now $\varphi_1 = \pi$. This location has traveled passed the pole and is on the opposite side of the sphere producing light polarized in the x direction. In this case the variable waveplate is acting like a half waveplate, which is why linearly polarized light is produced again, but perpendicular to the input polarization state.

In Figure 6.5 the linearly polarized light is horizontally polarized and perpendicular to Case 1. Complete darkness is achieved for a horizontal input with a vertical analyzer (a) and the highest intensity is displayed with the horizontal analyzer (c). The explanation for the intensities is the same as for Case 1.

![Figure 6.5: Horizontally polarized beam through different analyzer polarizations depicted by arrows.](Image)

6.1.4 Case 4: $\varphi_1 = 3\pi/2$ Produces Left Circular Polarization State

Case 4 in Figure 6.2 increases the phase difference even farther and returns to circularly polarized light, but in the opposite orientation. Here $\varphi_1 = 3\pi/2$ which is the equivalent to a quarter waveplate from Case 2 but oriented on the opposite axis. Due to this behavior, the results for Case 4 in Figure 6.6 are the opposite as Case 2 and left circularly polarized light is produced. For left circularly polarized light the highest intensity through an analyzer is produced with a left circular analyzer (e) and the dimmest intensity is produced.
when the right circular analyzer is used (f). The linear polarizations (a)-(d) are approximately the same intensity. Again, the explanation for the intensity distribution is the same as it was for Case 2.

![Images of polarized beams through analyzers](image)

**Figure 6.6**: Left circularly polarized beam through different analyzer polarizations depicted by arrows.

After Case 4 we returned to the original starting state of vertical polarization and signifies traveling a full loop on the longitude back to the original starting state. The variable waveplate can adjust the phase beyond $2\pi$, but the states produced would be the same as the previous cases. In Figure 6.7 below each state was achieved by traveling along a longitude line, and therefore only adjusting the phase difference of the polarization states.

![Diagram showing polarization states](image)

**Figure 6.7**: States created by changing $\phi_1$ and producing a phase change in the polarization state.

Next I will show the results when the rotator in Figure 6.1 is adjusted. Now the polarization state moves on the latitude direction along the equator as shown in Figure 6.8
Figure 6.8: States produced by rotating the orientation of the polarization state.

The rotator measurements are next which should rotate the beam around the equatorial points to obtain all linear polarization states, but polarized in different directions. The rotator is now the result of changing $\varphi_2$ in Figure 6.1. For the rotator, I am starting at the input polarization again, so there is no rotation occurring therefore Case 5, or the first state for the rotator is again the original input polarization state. The rotator changes the orientation of the axis of the beam, and if the starting point in Case 5 is vertical polarization then the rotator will rotate around the equator as seen in Figure 6.8.

6.1.5 Case 5: $\varphi_2 = 0$ Produces Vertical Polarization State

Case 5 in Figure 6.8 is the state when no rotation occurs and $\varphi_2 = 0$. The results produce in Figure 6.9 are the same as Case 1 in Figure 6.3 which is linearly polarized light in the vertical direction, and there is only a difference in the intensity of the horizontal analyzer (c).

Figure 6.9: Vertically polarized beam through different analyzer polarizations depicted by arrows.
6.1.6 Case 6: $\phi_2 = \pi/2$ Produces Negative $\pi/4$ Polarization State

Case 6 in Figure 6.8 is the result when $\phi_2 = \pi/2$ and the beam has been rotated around the equator by $\pi/2$. This produces a state oriented at $-\pi/4$. For the purpose of this experiment, positive $\pi/4$ is to be recognized as upward to the right or down the left. On a normal $x$, $y$ plane it corresponds to $\pi/4$ above the $x$ axis. Minus $\pi/4$ is therefore the opposite, and is $\pi/4$ above the negative $x$ axis or $\pi/4$ below the positive $x$ axis. Results for Case 6 shown in Figure 6.10 should produce the highest intensity when a $-\pi/4$ analyzer is present (d) and the darkest intensity when a $+\pi/4$ analyzer is present (b). The vertical and horizontal analyzers (a), (c) are dimmer as well as the circular analyzers (e), (f). The reasons for this are the same as the previous cases. Light oriented at $\pi/4$ radians has both an $x$ and $y$ component, so the analyzer for the $x$ and $y$ directions (a) and (c) only detect the one component and block the other half. The circular analyzers are dimmer because, as mentioned previously, the linear polarization is made of right and left circular components. Only one component is detected by the analyzer and the other is blocked causing the decrease in intensity.

![Images of polarization states](image)

Figure 6.10: Minus $\pi/4$ radians polarized beam through different analyzer polarizations depicted by arrows.

6.1.7 Case 7: $\phi_2 = \pi$ Produces Horizontal Polarization State

As the rotator is adjusted farther, the polarization state is perpendicular to the input beam as shown in Figure 6.8. When $\phi_2 = \pi$ the horizontal polarization state is produced. This shows that Case 3 and changing the phase is not the only way to achieve horizontal polarization. The results in Figure 6.11 should be the same as Case 3 in Figure 6.5 as well.
with the brightest output occurring for the horizontal analyzer (c), and the beam being completely blocked with a vertical analyzer (a).

![Figure 6.11: Horizontal polarized beam through different analyzer polarizations depicted by arrows.](image)

**6.1.8 Case 8: \( \varphi_2 = 3\pi/2 \) Produces Positive \( \pi/4 \) Polarization State**

Case 8 in Figure 6.8 is another state in between vertical and horizontal, but perpendicular to Case 6 and produces light oriented at \(+\pi/4\) radians. When \( \varphi_2 = 3\pi/2 \) the \(+\pi/4\) slanted polarization state is produced. In a similar fashion as the other linear states, the results in Figure 6.12 shown that the highest intensity occurs with a \(+\pi/4\) analyzer (b) and the lowest, or complete blockage occurs with the \(-\pi/4\) analyzer (d). The vertical (a), horizontal (c), left and right circular (e), (f) should be dimmer intensities, but some light is still let through.

![Figure 6.12: Positive \( \pi/4 \) radians polarized beam through different analyzer polarizations depicted by arrows.](image)

After the \(+\pi/4\) slanted polarization the rotator returns to vertically polarized light at \( \varphi_2 = 2\pi \) completing a full circle around the equator. As seen below in Figure 6.13 the 4 states produced in case 5-8 are rotating around the equator by adjusting \( \varphi_2 \).
The images above have allowed me to create the 6 important polarization states on the Poincaré sphere. I first traveled along the longitude and changed the polarization by changing $\varphi_1$ in Figure 6.2 from vertical to right circular to horizontal and finally to left circular. After that I traveled along the latitude at the equator by adjusting $\varphi_2$ in Figure 6.8 and produced vertical to $-\pi/4$ to horizontal to $+\pi/4$. Combining the two paths of motion on the sphere, any polarization state can be reached by changing $\varphi_1$ and $\varphi_2$.

Apart from the elliptical polarizations, which are hard to qualitatively analyze, I have been able to achieve the two circularly poles, and the four points around the equator, using only two devices with minimal adjustments to the system. Rather than manually adding in a quarter waveplate to achieve the poles, or changing the input polarization to achieve the equator coordinates, I can adjust the variable phase shifters without having to completely realign a new device into system. This optical system is a very simple system that allows for significant changes to the polarization state. Before moving on to more complex states, I wanted to perform one more test to make sure that the polarizations states are behaving properly in between the states found above.

While the images are evidence that the variable waveplate and the rotator are working, I wanted more quantitative evidence that they are behaving as expected. I switched the camera out for a detector to measure the intensity of the output for each polarization and graphed them accordingly. The analyzer was varied by $\pi/8$ for $2\pi$ radians to obtain data for the states in between those photographed. Normalized results are shown in Figure 6.14. For
the linear plane wave polarizations, I expect the peak intensity to move depending on the input polarization. I graphed the normalized linear polarizations to compare them together.

**Figure 6.14: Graph of normalized intensity for plane waves on zero order Poincaré sphere.**

From Figure 6.14, I can see that the intensity maximum is moving as the input plane of polarization moves, which is as predicted. A vertical maximum is the same angle as a horizontal minimum, which is why a horizontal analyzer completely blocks vertically polarized light. Now that it has been confirmed that optical system can be used to move around the zero order Poincaré sphere I want to apply it to the higher order polarization states, but first need to understand how the states differ from the zero order, and if this system can be applied to more complex states.
CHAPTER 7

FIRST ORDER POINCARÉ SPHERE

7.1 First Order Polarization States

An important characteristic of the zero order sphere is its spatial uniformity. The polarization of the beam is uniform throughout the entire beam. This characterization was proven using polarizers to analyze the beam which was done for the zero order sphere in Chapter 6. Any change in the polarization of the beam through the analyzer will be consistent for all points in the beam as observed. This is not necessarily true for all types of polarization states. There are new polarization states that are spatially variant in which the polarization is no longer uniform. The assumption that the polarization at one point in the beam is the same at all points is no longer valid and the state will not produce results like the zero order sphere. The polarization states that behave this way are defined as higher order polarization states which have their own Poincaré spheres that are similar to the zero order sphere but are spatially variant. The higher order states will be discussed now and the difference between the first order Poincaré sphere in Figure 7.1 and the zero order Poincaré sphere will become apparent.
Figure 7.1: Poincaré sphere for $l = +1$. Equator points represent radially to azimuthal translation. Poles are circular vortexes.\textsuperscript{11}

The higher order polarization states that lie on the equator of the first order Poincaré sphere are called radial, azimuthal, and spiral as shown in Figure 7.2. These are known as cylindrical vector beams.

![Poincaré Sphere Diagram](image)

(a) Radial, (b) Azimuthal, (c) $+\pi/4$ Spiral polarization states.

Figure 7.2: Radial (a), Azimuthal (b) and $+\pi/4$ Spiral (c) polarization states.\textsuperscript{13}

Figure 7.2a shows radially polarized light where the polarization directions are characteristically directed outward. The name of the state is due to its direction of radiating outward from a central location. This causes for vertical polarization along the $y$ axis, horizontal polarization along the $x$ axis and $+\pi/4$ and $-\pi/4$ polarizations in between the $x$ and $y$ axes. All of the states were present as individual states for the zero order now exist all at
once in one beam at different locations in the beam. Radially polarized light has a dark spot in the center because the polarization states are directed outward away from the center. This is known as a polarization singularity.

Azimuthally polarized light shown in Figure 7.2b is similar to radially polarized light in that the center still has the singularity, but the polarization is no longer radiating outward, but around the center. This produces horizontally polarized light on the y axis and vertically polarized light on the x axis. It has all the linear zero order polarizations in the beam, but now in the opposite locations as radially polarized light. If radially polarized light is the higher order equivalent of vertically polarized light, then azimuthal is the higher order equivalent of horizontally polarized light.

The final state above in Figure 7.2c is one of the spiral polarization states that corresponds to the higher order $+\pi/4$ slant. The $-\pi/4$ spiral is flipping the state above about its y axis to flip the direction of the states. In the same respect as the radial and azimuthal beams, each zero order linear polarization can be found in the beam, but on different axes then the other two states. The spatially variant states all exist on a Poincaré sphere as well in Figure 7.1. The relationship between the polarization states on the Poincaré sphere are the same for any polarization state, since the latitude and longitude of the Poincaré sphere are caused by the same factors, the angle of rotation $\phi$ and the ellipticity of the beam $\theta$. The same changes in the beam should occur regardless of the type of polarization state.

From Figure 7.1 it is seen that radial and spiral, and therefore azimuthal all exist on the equator of the beam. Starting from one of these states and traveling along the longitude introduces ellipticity in the beam and produces circular states on the poles. Recall the zero order sphere from Chapter 3 seen in Figure 7.3. From the zero order sphere, the position on the sphere for vertical polarization is the same as radial, and the same can be said for the slant and the spiral polarization states. Therefore the linear polarization states on the poles correspond to the vector beams on the first order sphere.
Figure 7.3: Zero order Poincaré sphere with latitude and longitude coordinates.\textsuperscript{11}

What makes the first order states significantly different than the zero order is that they do not have the same polarization throughout the beam. The polarization states are known as spatially variant polarizations and this means that a linear analyzer cannot block an entire beam, but only a portion of it because only a portion of the beam is polarized in that direction. The polarization states are difficult to produce naturally, but need a special device to generate them which will be discussed in Chapter 8. First I will explain how these states are represented in Jones vectors.

To create the first order sphere, an additional phase component is produced that corresponds to the angular momentum. Previously we have no orbital angular momentum in the zero order Poincaré sphere, which produces uniform polarization states. When the orbital angular momentum $\ell h$ is present, there is a helical phasing which induces an optical vortex. An optical vortex is such that there is a vortex at the center of the beam devoid of any light as seen in the center of the beams in Figure 7.2. This vortex is present in all the states found on the sphere in Figure 7.1. This optical vortex is known as the azimuthal phase factor $e^{2i\alpha}$. The polarizations on the first order sphere are spatially variant but mirrored across the vertical or horizontal axis. In the first order case, the polarization state at each point is related to one another by a symmetrical rotation. The rotation follows the azimuth angle $\alpha = \arctan(y/x)$. A new vector for the spatially variant states can be produced for $(\alpha + \phi)$. This will produce an equation similar to Equation (3.12) but includes the additional azimuth angle $\alpha$: 
\[ E(\phi, \theta) = \begin{bmatrix} \cos(\alpha + \phi) \cos \theta - i \sin(\alpha + \phi) \sin \theta \\ \sin(\alpha + \phi) \cos \theta + i \cos(\alpha + \phi) \sin \theta \end{bmatrix}. \] (7.1)

In this equation \( \phi \) is the angle of axis rotation and \( \theta \) is the ellipticity coordinate.

This equation now describes the higher order Poincaré sphere polarization states. Radial and azimuthal polarizations exist when \( \phi = 0 \) and \( \pi/2 \) and \( \theta = 0 \), which is the same latitude and longitude coordinates of the zero order, but with a different polarization output. This gives them the Jones vector forms as:

\[ E_{\text{rad}} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix}, \] (7.2)
\[ E_{\text{az}} = \begin{bmatrix} -\sin \alpha \\ \cos \alpha \end{bmatrix}. \] (7.3)

The two polarization states in between radial and azimuthal correspond to the \( \pm \pi/4 \) linear polarization spirals which similarly require an additional \( \pi/4 \) added to the azimuthal angle:

\[ E_{\text{spiral}} = \begin{bmatrix} \cos \left( \alpha \pm \frac{\pi}{4} \right) \\ \sin \left( \alpha \pm \frac{\pi}{4} \right) \end{bmatrix}. \] (7.4)

The final two points on the sphere are the poles which are when \( \theta = \pm \pi/2 \). These are circularly polarization states with an additional \( e^{il\alpha} \) where \( l = 1 \) and,

\[ E_{\text{CP}} = e^{il\alpha} \begin{bmatrix} 1 \\ \pm i \end{bmatrix}. \] (7.5)

The matrix to generate the spatially variant states will be a rotation matrix, but in terms of the azimuth angle \( \alpha \):

\[ M = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}. \] (7.6)

A device to produce this matrix is called a radial polarization converter and will be covered in Chapter 8.
Similarities can be seen between the first order case and the zero order case, but with the additional azimuthal phase factor in front. The phase in this case doesn’t rotate the polarization, but creates the vortex in the center of the beam because it is the orbital angular momentum, and not the spin. The spin angular momentum produces circular polarization, and the orbital angular momentum produces the vortex.

The Jones vectors help described the polarization of the state, but the more complex states there are, the more difficult the Jones vectors are to find. For the purposes of this experiment, the Jones vectors are sufficient to understanding the polarization states, but Stokes Parameters are another common way to describe the polarization states which lend themselves the ability to calculate intensity as well as describe the ellipticity of any polarizations between the poles and the equator. More can be seen about the other method of analyzing the polarization states in Appendix A.

The order number of the Poincaré sphere is determined by the angular momentum of light as mentioned previously. Quantitatively, this has a large influence in the Stoke’s Parameters (Appendix A) but in the Jones Matrices, it is included as part of the azimuthal phase factor. In the first order case, \( l = 1 \). For the equatorial polarizations, the actual vectors for the equator after converting \( e^{i\alpha} = \cos(l\alpha) - i\sin(l\alpha) \) are given by

\[
E_{\phi=0} = \begin{bmatrix} \cos l\alpha \\ \sin l\alpha \end{bmatrix} \text{ and } E_{\phi=\frac{\pi}{2}} = \begin{bmatrix} -\sin l\alpha \\ \cos l\alpha \end{bmatrix}.
\] (7.7)

The \( l = -1 \) Poincaré sphere results from a \( \pi \) phase shift from the \( l = +1 \) case. Due to this nature, it should be expected that the polarization on the poles will be opposite to the \( l = +1 \) case, as well as a reverse in rotational direction around the equator.
Figure 7.4: Poincaré sphere for $l = -1$. Equator is $\pi$ radial to $\pi$ azimuthal translation and poles are opposite circular vortexes than +1 sphere.\textsuperscript{11}

Figure 7.4 is the $-1$ Poincaré sphere. The polarization states are similar to the +1 sphere, but with a $\pi$ phase shift in the vertical direction. They resemble the +1 sphere in that they are still spatially variant, but the vector directions are different. The circular poles are flipped and the linear states on the principle axes of the phase shift are flipped $\pi$ radians and the states at $\pi/4$ from the principle axes are rotated $\pi/2$ radians. The elliptically polarized beams have their axes in the same direction as the linear state at the equator for each longitude line. The states on the $-1$ Poincaré sphere will be detailed more when data for the $-1$ sphere is taken.

To generate these polarization states, a radial polarization converter is used that can change either vertical or horizontal input into radial or azimuthal polarization states. This device is key to producing the higher order Poincaré sphere states and will be discussed next.
CHAPTER 8

RADIAL POLARIZATION CONVERTER

To create the states described in the previous chapter, I am using a device developed by ARCoptix called the radial polarization converter.

Figure 8.1: ARCoptix radial polarization converter.14

The radial polarization converter relies on a liquid crystal aligned so that the orientation of the linearly polarized beam is rotated locally for each molecule of the liquid crystal. Inside the ARCoptix device the entrance and exit plates of the cell are linearly and circularly rubbed respectively. The direction of the linear rubbing on the entrance determines the cell axis. Each molecule chain of the liquid crystal inside the cell is characterized by a twist angle, which is the angle between the molecule orientation at the entrance and exit plates. The orientation of the input beam determines whether the output is radial or azimuthally polarized light based on whether it the input is parallel or perpendicular to the cell axis. When a linearly polarized beam enters, each point in the beam interacts with a molecule with a different twist angle and each segment is rotated by a different angle. In this case the rotation is symmetric about the $x$ axis and radial or azimuthal polarization is
produced. The symmetry in the polarization state is caused by the direction of the rotation. The top half of the cell rotates clockwise and the lower half rotates counterclockwise. A dark line along the x axis of the cell is produced caused by the different rotation sense in the two parts. The defect line, and amount of rotation can be seen in Figure 8.2 below.

![Figure 8.2: Liquid crystal alignment with defect line present due to different rotation directions in upper and lower halves. Arrow represents input direction.](image)

In addition to the theta-cell which houses the liquid crystal rotator the ARCoptix has a twisted nematic (TN) cell that allows for switching between radial and azimuthal polarizations without changing the input polarization state. The TN cell is essentially an achromatic half-wave plate that rotates each polarization in the beam by $\pi/2$ when a voltage is applied.

A third element is added to the ARCoptix, which is an additional liquid crystal phase retarder that is placed before the theta cell. The purpose of the phase retarder is to correct a $\pi$ phase step in the center of the beam. This phase step is was produces the defect line caused by the opposite directions of rotation in the upper and lower halves. The phase retarder is also driven by a voltage like the TN cell, but it instead provides a tunable phase delay between the two halves. When the voltage applied for the appropriate wavelength the defect line disappears. The phase retarder acts like a half wave plate when at the correct voltage. The liquid crystal molecules can only rotate up to $\pi/2$, so between the two halves there is a $\pi$
phase shift. The phase retarder only covers half of the cell, and therefore applies a $\pi$ phase shift on one have to produce the same phase on both halves. By applying a voltage to the phase retarder, the dark defect line can be removed or lightened, and the beam is homogenized. The difference between the two states can be seen below in Figure 8.3.

![Figure 8.3: Output from ARCoptix with $V_1=0.00V$ (a) and with $V_1=1.4V$ to remove the defect line (b).](image)

After adding in the phase retarder voltage, a different kind of radial polarization state is produced known as pseudo radially polarized light. The $\pi$ phase shift that the $\theta$-cell produces causes pure radial polarization, which also causes the defect line because the vertical components of each localize vector are opposite across the defect line. The phase retarder applies an additional half waveplate to the lower half to produce the pseudo radial polarization states. The direction of the vectors are more uniform, and so the defect line disappears. The optical vortex is present in both cases, because not all the light in the pseudo case in Figure 8.4a crosses through the center, but in Figure 8.3 the vortex is larger in case (a) where pure radial is being produced. In most cases the pseudo-radially polarization is still considered radially polarized light, but on the Poincaré sphere it is not the appropriate state. The setting in Figure 8.3 (b) will not be used for this experiment due to the property of the phase retarder. Therefore the phase compensator will not have a voltage and $V_1=0.00V$. As stated previously when $V_1=1.4V$ the phase retarder acts like a half waveplate on only one half of the cell. This has little effect on the linear polarization states, but when circular enters the ARCoptix then it produces right circularly polarized light on one half and left circularly
polarized light on the other. The results from this experiment are in Appendix C and determined that the beam will not be homogenized with the ARCoptix.

![Figure 8.4: Pseudo radial (a) and pure radial (b) polarization states.](image)

The complete system that the ARCoptix uses is seen below in Figure 8.5.

![Figure 8.5: ARCoptix Radial Polarization converter with TN cell to rotate between radial and azimuthal polarizations and the variable phase shifter to homogenize the beam.](image)

In Figure 8.5 only the $\theta$-cell produces the radial or azimuthal polarization states. As previously discussed the polarization rotator switches between radial and azimuthal when the voltage is applied, and the variable phase shifter removes the defect line if desired when a different voltage is applied. For this experiment neither voltage settings will be used.
Figure 8.6: Front view of the liquid crystal molecule twist inside the cell. The line denotes defect line present due to different twist sense in upper and lower parts. The different inputs create different polarization states.

Just like any other device in an optical system, the ARCoptix can be described by a Jones matrix, which is how I will test if it will work with the rotator and variable wave plate in the system. The matrix can be derived through the input polarization. When vertically polarized light enters and no voltage is applied azimuthally polarized light is produced, and when horizontal enters radially polarized light is produced. Figure 8.6 demonstrates the results when a voltage is applied to the TN cell in which vertically polarized light produces radially polarized light. For the purpose of finding the Jones matrix for the radial polarization converter, it will be implied that no voltage settings are used. Recall the vectors for radially and azimuthally polarized light:

\[
E_{rad} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix} \quad \text{and} \quad E_{az} = \begin{bmatrix} -\sin \alpha \\ \cos \alpha \end{bmatrix}.
\] (8.1)

Using Equation (8.1) and the input polarization we can derive the matrix of the ARCoptix:

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix},
\] (8.2)
\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
\begin{pmatrix}
0 \\
1
\end{pmatrix}
= 
\begin{pmatrix}
-\sin \alpha \\
\cos \alpha
\end{pmatrix}
= 
\begin{pmatrix}
B \\
D
\end{pmatrix}.
\]

From this we can conclude that the matrix for the ARCoptix is:

\[
M_{ARC} = \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}.
\]

This is the matrix described in Chapter 7 in Equation (7.5) on how to produce radial and azimuthally polarized light, and this is just a rotation matrix about the azimuthal phase angle \(\alpha\). This logically makes sense because each segment of radial or azimuthally light is just the input rotated by an angle. Since the ARCoptix can be described by a rotation matrix, the rotator will have a linear relationship on the polarizations, and the variable waveplate will cause the same phase changes as it did for the zero order Poincaré sphere. Just as I was able to map the entire zero order sphere, I should be able to do the same to the positive and negative first order spheres, but I need to be careful to remember than the polarization states on the first order Poincaré sphere are spatial variant and will not behave exactly like the zero order polarization states.

Previous results have shown how radially polarized light is manipulated through different polarizers and waveplates.\(^{15}\) From the previous results I can see that the changes to \(\phi_1\) and \(\phi_2\) need to be applied before the optical vortex is created. This should not cause issues because the ARCoptix is designed to allow different input polarizations through. Figure 8.7 below shows how each individual vector interacts with a quarter waveplate, and how those on the same axis as the waveplate are unaffected.

\[\text{Input radial} \quad \text{QWP} \quad \text{Transformed vector beam}\]

Figure 8.7: Manipulation of a radially polarized beam.\(^{16}\)
Before I begin manipulating \( \varphi_1 \) and \( \varphi_2 \) I review the optical system from the zero order to confirm that it will not create states like the state in Figure 8.7 which is not a state present on the first order Poincaré sphere.
CHAPTER 9

OPTICAL SYSTEM FOR THE FIRST ORDER

The optical system used for the first order spheres is the same system as the zero order but with the ARCoptix radial polarization converter to produce the vector beams. The higher order Poincaré spheres may have complex polarization states, but changing $\varphi_1$ and $\varphi_2$ should produce the same effects as they did for the zero order. To change the latitude, the polarization state is rotated, and to change the longitude the phase difference of the polarization state is changed. For the zero order we proved that the system worked to make the changes to the latitude and longitude, and therefore the same optical system can be used for the first order Poincaré spheres seen in Figure 9.1 below.

Figure 9.1: Diagram of setup. The RPC is the ARCoptix radial polarization converter.
Now that I have the means to create radially and azimuthally polarized light, I need add the ARCoptix to the zero order optical system. With the zero order, the order of the optical system did not matter due to the uniformity of the beam. With a spatially variant beam, it is not as simple. States that are not on the sphere, like those shown in Figure 8.7 in Chapter 8 could be created from this same setup by changing the order of the system. The ARCoptix will be placed after the system from the zero order Poincaré sphere. As stated previously, the ARCoptix is designed so that different input polarizations produce different states, so any linear input should produce a first order equivalent on the sphere. Therefore it is better to manipulate a linear polarization state before the device then a vector beam state after.

To generate the $l = -1$ order, I will include a half wave plate at the end of the setup oriented at 0 radians to cause the $\pi$ phase shift needed to produce the $\pi$-radial and $\pi$-azimuthal polarizations. I will use the same camera as the zero order images to maintain image quality and beam size. The final image of my experimental setup is the same as the zero order, but with the radial polarization converter included.
CHAPTER 10

THE POSITIVE FIRST ORDER RESULTS

10.1 Positive first Order Images

The results for the positive first order Poincaré sphere follow the same pattern as the zero order Poincaré sphere results. First I will use the variable phase shifter to change \( \varphi_1 \) and change the ellipticity. From Figure 10.1 below I should be able to produce the circular poles on the states and radial and azimuthal polarization states, which are all achieved by traveling along a longitude line. After producing those results I will vary \( \varphi_2 \) to rotate the polarization state around the equator. Each vector beam in the polarization state will be rotated by the same angle, and therefore I will be able to produce radial, positive \( \pi/4 \) spiral, azimuthal and negative \( \pi/4 \) spiral.

Figure 10.1: First order Poincaré sphere polarization states.
The experimental methods follow the same format as the zero order Poincaré sphere in Chapter 6. Cases 1-4 I traveled on the longitude by changing the phase shift $\phi_1$ in Figure 10.2 and produced the states shown in figure 10.3.

Figure 10.2: Optical system for the first order Poincaré sphere.

Figure 10.3: The positive first order Poincaré sphere coordinates produced when changing $\phi_1$. 
10.1.1 Case 1: $\varphi_1 = 0$ Produces Azimuthal Polarization State

Case 1 in Figure 10.3 is created when $\varphi_1 = 0$ in Figure 10.2 and will produce the azimuthal polarization state. This is due to no voltage being applied to the TN cell of the ARCoptix, so a beam polarized in the y direction will produce azimuthally polarized light. Results are shown in Figure 10.4.

Figure 10.4(a) shows the full intensity of the beam un-obstructed, and the bright spots of the other beams are of the same intensity as the pure beam which indicates that all of polarization state is let through the analyzer in those positions. The azimuthal polarization state behaves like the image above where the polarization on an axis is perpendicular to that axis. The horizontal polarization states occur on the vertical axis and therefore a horizontal analyzer (d) produces light in the vertical direction. The same is true for the other linear states as well. The vertical analyzer (b) reveals the vertically polarized light on the horizontal axis, the $+\pi/4$ analyzer (c) is pointing in the $-\pi/4$ direction, and the $-\pi/4$ analyzer (e) is pointing on the $+\pi/4$ axis. The circular analyzers (f), (g), should produce the same beam shape as (a) but with a lower intensity.

![Figure 10.4: Azimuthally polarized light through different analyzers when $\varphi_1=0$.](image)

10.1.2 Case 2: $\varphi_1 = \pi/2$ Produces Right Circular Polarization State

Case 2 in Figure 10.3 produces right circular polarization state. The first order Poincaré sphere changes its ellipticity in the opposite direction of the zero order case, therefore instead of producing left circularly polarized light when $\varphi_1 = \pi/2$ it produces right circularly polarized light. The results expected for this state in Figure 10.5 are very similar to the zero order case. The brightest beam occurs when the right circular analyzer (g) allows the entire beam to pass through to the camera, and the weakest beam is when the left circular
analyzer (f) blocks the entire beam. The linear analyzers (b)-(e) will again display a decreased intensity, but retain the shape of the original beam.

Figure 10.5: Right circularly polarized light through different analyzers when $\varphi_1 = \pi/2$.

10.1.3 Case 3: $\varphi_1 = \pi$ Produces Radial Polarization State

Case 3 in Figure 10.3 is when $\varphi_1 = \pi$ and the radial polarization state is produced, indicating that Case 3 is on the opposite side of the sphere to Case 1. Results shown in Figure 10.6 for radially polarized light are expected to produce that the vertical analyzer (b) allows the vertically polarized vector beams in the y axis through and blocks the horizontal polarization on the x axis. The reverse is true for the horizontal analyzer (d) which illuminates the polarization on the x-axis and blocks the y axis polarization states. The $+\pi/4$ (c) and $-\pi/4$ (e) analyzers behave similarly and allow the $+\pi/4$ and $-\pi/4$ through and block the perpendicular axis respectively. The circular analyzers behave exactly as they did for the zero order sphere and exhibit a dimmer intensity, but some from every state to pass. Again, this is due to linearly polarized light, which is what each localized vector is, being a linear combination of right and left circular polarizations.

Figure 10.6: Radially polarized light through different analyzers when $\varphi_1=\pi$.

10.1.4 Case 4: $\varphi_1 = 3\pi/2$ Produces Left Circular Polarization State

Case 4 in Figure 10.3 is when $\varphi_1 = 3\pi/2$ and the left circular polarization state is produced as shown in the results in Figure 10.7. The beam is circularly polarized so there
should be no change in the beam distribution when the linear analyzers are utilized, which is true in (b)-(e) apart from a lower intensity, which is expected. The left circularly polarized light should produce a higher intensity. The final image is the right circular analyzer, which should not allow any left circularly polarized light through, which is displayed in (g) with a completely dark signal.

Figure 10.7: Left circularly polarized light through different analyzers when $\varphi_1 = 3\pi/2$.

The next case is when $\varphi_1 = 2\pi$ and the state returns to the azimuthal polarization state. After completing the change to the longitude without error, I moved on to the rotator in Figure 10.2 and now study changes along the equator as shown in Figure 10.8.

Figure 10.8: Positive first order Poincaré sphere coordinates produced by changing $\varphi_1$.

10.1.5 Case 5: $\varphi_2 = 0$ Produces Azimuthal Polarization State

Case 5 in Figure 10.8 is the initial output of the azimuthal polarization state. The results in Figure 10.9 here should be the same as Case 1 in Figure 10.4, since it is the same starting state. From here the phase shifter inside the rotator will be adjusted to rotate the
orientation of the polarization states to produce states around the equator as seen in Figure 10.8.

Figure 10.9: Azimuthally polarized light through different analyzers when $\varphi_2 = 0$.

10.1.6 Case 6: $\varphi_2 = \pi/2$ Produces $+\pi/4$
Spiral Polarization

The $+\pi/4$ spiral polarization state occurs when $\varphi_2 = \pi/2$ around the equator in Figure 10.8. By looking at the polarization directions in the results in Figure 10.10, the $+\pi/4$ analyzer should illuminate the y axis. Figure 10.9c below agrees with this statement. This also means that the $-\pi/4$ slanted analyzer will illuminate the x axis as seen in Figure 10.10e. The vertical and horizontal polarization states occur at $-\pi/4$ (b) and $+\pi/4$ (d) respectively. The circular analyzers (f), (g) again don’t change the beam shape significantly and display a slight decrease in intensity.

Figure 10.10: $+\pi/4$ spirally polarized light through different analyzers when $\varphi_2 = \pi/2$.

10.1.7 Case 7: $\varphi_2 = \pi$ Produces Radial Polarization State

The result in Case 7 produces the radial polarization state when $\varphi_2 = \pi$ around the equator in Figure 10.8. In the results in Figure 10.11 the linear analyzers produce vertically polarized light on the y axis (b), and horizontally polarized light on the x axis (d). In a similar fashion the $+\pi/4$ slant is on the $+\pi/4$ axis (c) and the $-\pi/4$ slant is on the $-\pi/4$ axis (e). The circular analyzers (f), (g) should not change the shape of the beam drastically and only undergo a decrease in intensity.
10.1.8 Case 8: $\varphi_2 = 3\pi/2$ Produces $-\pi/4$

Spiral Polarization State

When $\varphi_2 = 3\pi/2$ around the equator the opposite spiral polarization state is achieved where the direction is the $-\pi/4$ direction in Figure 10.8. In the results shown in Figure 10.12 this state should produce the $-\pi/4$ vectors on the $y$ axis (e), and so the $+\pi/4$ vectors are on the $x$ axis (c). This also puts the vertical and horizontal polarizations on the $+\pi/4$ and $-\pi/4$ axes (b), (d) respectively. Like the previous states on the equator the circular analyzers only display a decrease in intensity.

The next case occurs $\varphi_2 = 2\pi$ and a rotation around the $x$ axis is completed. All of the states labeled in Figure 10.8 were achieved only by changing $\varphi_2$ in Figure 10.2.

From the results above, I was able achieve all points along a longitude line starting at azimuthal, to right circular, to radial, to left circular and returning to the start. I was also able to travel along a latitude line starting at radial to $-\pi/4$ to azimuthal, to $+\pi/4$ and back to radial. The motion across the sphere was achieved with only two devices, a variable waveplate and a rotator. Using the two devices, I could reach any point on the first order Poincaré sphere first by travelling up or down the longitude, and then to either side along the latitude.

Applying the waveplate and rotator to the ARCoptix provides a very elegant system for understanding and exploring the higher order Poincaré sphere states. The system required very little modification and the quality of the results were very strong.
After achieving successful results with the +1 sphere I used the same technique and equipment for the −1 order with the addition of a half waveplate after the radial polarization converter. The results can be seen in the next chapter.
CHAPTER 11

THE NEGATIVE FIRST ORDER RESULTS

11.1 Negative first Order Images

The same transformations made to the positive first order Poincaré sphere can also be done to the negative first order polarization states as well. Before continuing, the differences between the polarization states needs to be characterized so I understand what the expected outcome should be for each analyzer.

Recall the first order Poincaré sphere from Chapter 7 repeated in Figure 11.1 below:

![Figure 11.1: Poincaré sphere when \( l = +1 \). The equator points represent radially to azimuthal translation and the poles are circular vortexes.](image)

By contrast, Figure 11.2 below shows the \(-1\) Poincaré sphere.
The $-1$ Poincaré sphere is similar to the $+1$ Poincaré sphere in that they are still spatially variant beams, and still maintain the optical vortex. By comparing the two, it can be seen that the poles have been flipped, and the equatorial points are augmented, but only in certain directions. From my experiences with ARCoptix and the half circular poles (Appendix C), I learned that a half wave plate flips the circular polarization states, which would achieve the correct poles for the $-1$ sphere, and the changes to the equatorial points are also due to a half waveplate. Therefore to produce the $-1$ Poincaré sphere polarization states I will add a half waveplate at 0 radians (not shown) after the ARCoptix radial polarization converter in the optical setup in Figure 11.3 below.
When I compared the measurements of the $-1$ to the $+1$ order, I noticed differences in both measurements of $\varphi_1$ and $\varphi_2$. The poles are reversed, as expected and the measurements move in the opposite direction. For the change in the system that produced radial in the $+1$ order, the $-1$ achieved the $\pi$-azimuthal. With the rotator, the starting point was the same, but the minus 1 order rotated around the opposite direction for the same measurements. I then proceeded to take the photos again, making sure that the proper polarization state was occurring at each setting.

From Figure 11.4 I can observer that the $x$ and $y$ axis are still vertically and horizontal oriented, but the $-\pi/4$ and $+\pi/4$ axis in the $\pi$-radial state (b) are perpendicular to the results from the radial (a). This is where the image results will vary from the results in Chapter 10. This relationship is true for all the $+1$ states and the $-1$ equivalent states,
For the negative first order Poincaré sphere, the settings that produce azimuthal polarization produce the \( \pi \)-radial, which will be the starting point for the phase and rotator measurements. The experimental methods are again the same as it was for the zero order and the +1 order. Cases 1-4 are the result of changing \( \phi_1 \) in Figure 11.3 and will travel along the longitude above and below the equator as seen in Figure 11.5.
11.1.1 Case 1: $\phi_1 = 0$ Produces $\pi$-Radial Polarization State

Case 1 in Figure 11.5 for the phase shifter is the $\pi$-radial polarization state, which is similar to radial polarization except for some vector states are in the perpendicular direction as described from Figure 11.4.

From the results in Figure 11.6 of the polarization directions, I can conclude that I expect the vertical analyzer (b) to illuminate the vertical direction, and the horizontal analyzer (d) to produce light in the horizontal direction. The output that differs from the original radial polarization is the other 2 linear polarizations. On the positive first order sphere the radial polarization had $+\pi/4$ slanted polarization on the $+\pi/4$ axis and the $-\pi/4$ polarization on the $-\pi/4$ axis. In the Case 1 it is reversed.

By looking at the images, I can see that the output does indeed follow the expected outcome. The vertical analyzer (b) is on the $y$ axis, the $+\pi/4$ analyzer (c) illuminates the $-\pi/4$ axis, the horizontal analyzer (d) is the $x$ axis, and the $-\pi/4$ analyzer (e) is in the $+\pi/4$ axis. Finally, because this polarization state is only made up of linear polarizations the circular analyzers (f), (g) behave just like they did for the vector states and display a dimmer intensity than the full unobstructed beam (a).

![Figure 11.6: $\pi$–radially polarized light from $-1$ order Poincaré sphere when $\phi_1 = 0$.](image)

11.1.2 Case 2: $\phi_1 = \pi/2$ Produces Right Circular Polarization State

Case 2 in Figure 11.5 is the right circular polarization state pole when $\phi_1 = \pi/2$ for the negative first order sphere and is the same state as the positive first order, except located on the opposite poles. In the positive first order sphere after radially polarized light the phase produced left circular polarization, but in this case it produces right circular. The results below in Figure 11.7 exhibit the same behavior previously seen. The left circular analyzer
blocks the beam (f), and the right circular analyzer produces a high intensity beam (g). The
linear analyzers (b)-(e) only show a decreased intensity.

Figure 11.7: Right circularly polarized light produced from the negative first order
Poincaré sphere when \( \phi_1 = \pi/2 \).

11.1.3 Case 3: \( \phi_1 = \pi \) Produces \( \pi \)-
Azimuthal Polarization State
Case 3 in Figure 11.5 is when \( \phi_1 = \pi \) and the polarization state is perpendicular to the starting
state and produces the \( \pi \)-azimuthal polarization state. The results in Figure 11.8 show that
just like the \( \pi \)-radial the \( x \) and \( y \) axis will remain oriented in the same direction as the positive
first order case. Since this is an azimuthal state the horizontal analyzer will block the \( x \) axis
and allow the \( y \) axis light through (d). The vertical analyzer illuminates the \( x \) axis and blocks
the \( y \) axis (b). The \( +\pi/4 \) and \( -\pi/4 \) analyzers show the difference between the negative and
positive first order spheres. In the positive first order case the azimuthal state produce
vectors in the \( +\pi/4 \) and \( -\pi/4 \) directions on the \( -\pi/4 \) and \( +\pi/4 \) axes respectively. The \( \pi \)-
azimuthal does the opposite, the \( +\pi/4 \) slanted vector is on the \( +\pi/4 \) axis as shown in Figure
11.8c and the \( -\pi/4 \) slanted vector is on the \( -\pi/4 \) axis shown in 11.8e. The circular states (f)
and (g) characteristically show a decreased intensity but no change to the shape of the beam.

Figure 11.8: \( \pi \)-azimuthal polarization state produced from the negative first order
Poincaré sphere when \( \phi_1 = \pi \).
11.1.4 Case 4: $\varphi_1 = 3\pi/2$ Produces Left Circular Polarization State

Case 4 in Figure 11.5 is the left circular polarization state at $\varphi_1 = 3\pi/2$. This is the same state present on the positive first order sphere and the results in Figure 11.9 characteristically displays a high intensity when the left circular analyzer is present (f) and no intensity with the right circular analyzer (g). The linear analyzers (b)-(e) show a decreased intensity. The results are very familiar by this point.

Figure 11.9: Left circularly polarized light produced on the negative first order Poincaré sphere when $\varphi_1 = 3\pi/2$.

After the previous state the adjustments of $\varphi_1$ returns to $2\pi$ and the starting polarization of $\pi$-radial, indicating a complete loop around a longitude and achieving every point in Figure 11.5.

Next the phase inside the rotator $\varphi_2$ in Figure 11.3 is changed to rotate around the equator, starting with $\pi$-radial again and moving around to $+\pi/4$, $\pi$-azimuthal, $-\pi/4$ and returning to $\pi$-radial as described in Figure 11.10 below.
11.1.5 Case 5: $\phi_2 = 0$ Produces $\pi$-Radial Polarization State

Case 5 in Figure 11.10 is the same starting state as the phase shifter measurements. This polarization state is the same as Case 1 and is the start state for rotating around the equator. The results in Figure 11.11 exhibit the same behavior as Case 1 with vertical polarization in the $y$ direction (b), horizontal polarization in the $x$ direction (c), $+\pi/4$ polarization on the $-\pi/4$ axis (c), and $-\pi/4$ polarization on the $+\pi/4$ axis (e).

11.1.6 Case 6: $\phi_2 = \pi/2$ Produces Positive $\pi/4 - \pi$-spiral Polarization State

Case 6 in Figure 11.10 the $\pi$-spiral polarization state oriented at $+\pi/4$ that occurs when $\phi_2 = \pi/2$ around the equator. In the positive first order case the beam traveled from
radial to $-\pi/4$, which shows that the negative first order travels in the opposite direction around the sphere and follows the path set in Figure 11.10. This state is characterized by having the states on the $x$ and $y$ axes the same as the positive first order, but the other linear states are flipped. The results in Figure 11.12 below demonstrate that the $+\pi/4$ analyzer detects $+\pi/4$ linear polarizations on the $y$ axis (c) and the $-\pi/4$ analyzer detects $-\pi/4$ linear polarizations on the $x$ axis (e). The orientations have not changed in relation to the positive first order. The difference is the states on the $-\pi/4$ and $+\pi/4$ axes. The vertical polarization is now oriented perpendicular to its positive first order and is on the $+\pi/4$ axis (b). The same can be said for the horizontal polarization on the $-\pi/4$ axis (d). The circular states (f) and (g) behave normally and lower the intensity of the beam.

Figure 11.12: Positive $\pi/4$ $\pi$-spiral polarization states produce on negative first order Poincaré sphere when $\phi_2 = \pi/2$.

11.1.7 Case 7: $\phi_2 = \pi$ Produces $\pi-$ Azimuthal Polarization State

Case 7 in Figure 11.10 produces the same $\pi$-azimuthal polarization state found when changing $\phi_1$. In the rotation this state occurs when $\phi_2 = \pi$. Following the pattern set by the first two cases, Case 7 also maintains the same direction for the states on the $x$ and $y$ axis. The results in Figure 11.13 are the same as Case 3. The vertical analyzer (b) illuminates the $x$ axis and the horizontal analyzer (d) illuminates the $y$ axis. The other two remaining linear states are again flipped. The $+\pi/4$ analyzer (c) is in the $+\pi/4$ axis, which is opposite of the $+1$ Poincaré sphere (Figure 10.8), and the $-\pi/4$ analyzer (e) is in the $-\pi/4$ axis. The circular states (f), (g) still maintain the un-obstructed beam (a) shape, but with decreased intensity.
11.1.8 Case 8: \( \varphi_2 = \frac{3\pi}{2} \) Produces Negative \( \pi/4-\pi \)-Spiral Polarization State

Case 8 in Figure 11.10 is when \( \varphi_2 = \frac{3\pi}{2} \) and follows the same pattern as the ones before. The results in Figure 11.14 below show that the \( -\pi/4 \) analyzer (e) is on the y axis, \(+\pi/4 \) analyzer (c) is on the x axis and the other two states are flipped in comparison to the +1 Poincaré sphere. The vertical analyzer (b) is on the \( -\pi/4 \) axis and the horizontal analyzer (d) is on the \(+\pi/4 \) analyzer. The circular states (f) and (g) display a decreased intensity of the unblocked beam (a).

The next polarization is when \( \varphi_2 = 2\pi \) and it has returned to \( \pi \)-radial, completing a loop around the equator. All the states shown in Figure 11.10 were achieved. When combining this motion with the states produced by changing \( \varphi_1 \) the movement around the sphere is expansive.

I was again successfully able to generate all of the six cardinal polarization states. The addition of the half wave plate at zero radians is another simple adjustment to the system, which after adding it to the system requires no adjustments. I am able to generate 12 different polarization states, with a very simple optical system. The system will allow for generation of all of the first order Poincaré sphere coordinates for both the positive and
minus first orders. This system is very helpful for studying the behavior of the vector beam polarization states. The addition of the ARCoptix allows for simple generation of the first order states simply by turning a zero order homogenous polarization state into a vector beam. A half waveplate at the end of the system oriented at zero radians is another very easy modification to generate the negative first order vector beams. Using the system shows that the transformations on the zero order sphere can be easily applied to the first order states and the vector beam polarization states can be mapped onto the Poincaré sphere in a similar fashion to the zero order polarization states.
CHAPTER 12

CONCLUSION

This thesis summarizes the work in producing the first order Poincaré sphere coordinates using simple optical equipment. The research in this field is new and still developing. Such experiments involve manipulating the polarization state\textsuperscript{15} and generating the polarization states inside of a laser cavity.\textsuperscript{11} The previous experiments only produced either radial or azimuthally polarized light, with the inability to convert from one to the other easily, using a system that required rotating the entire axes of several waveplates. While the higher order Poincaré spheres can be produced, only the first order has strongly determined applications in science and industry, although there is still much to learn about the vector beam states.

In this thesis I have explained how to describe light from Maxwell’s equations, and how to write them in vector form using Jones vectors. The Jones vectors can be manipulated, and therefore the polarization can be changed using matrices that describe the changes caused by optical equipment. Using the Jones matrices I have shown how the matrix for a waveplate produces a phase change between the $x$ and $y$ components of the beam and produces circularly polarized light. A rotator system rotates the axis of the polarization allowing changes such as rotating from vertically polarized light to horizontally polarized light without a decrease in the intensity.

The system created for the zero order Poincaré sphere was successful in traveling along the longitude with the variable waveplate, along the latitude with the rotator. An important characteristic of the zero order sphere is its spatial uniformity. The polarization of the beam is uniform for all states on the zero order Poincaré sphere. The characteristic
uniformity is proven using analyzers. Any change in the polarization state occurs uniformly in the beam. The results for the zero order Poincaré sphere display this spatial uniformity.

I applied the same system to the positive first order Poincaré sphere by adding in a radial polarization converter which produces either radial or azimuthal polarization states. The polarization states are spatially variant in which the polarization direction changes based on the position in the beam area. The states in the first order Poincaré sphere are similar to the zero order sphere, but are spatially variant polarization states.

The ARCoptix radial polarization converter produces pure radial polarization when the voltage applied to the phase compensator is 0.00V, but a dark line is present in the results due to the y components of the vectors facing opposite directions on each half. The data was recorded with the line present. The spatial variance can be observed such that the linear analyzers do not cause the same change for the entire beam, resulting in bright spots along an axis in the beam. The same system was also applied to the negative first order sphere which is created by adding a half wave plate after the radial polarization converter. The results are also spatially variant.

The final conclusions of this thesis can be summarized in that the system of a variable waveplate and a rotator can be used to travel along the longitude and latitude of a Poincaré sphere of any order. The zero order sphere should produce linear plane wave polarization states characterized by spatial uniformity and the first order Poincaré spheres produce vector beams that are spatial variant. This system allows for very simple conversion from homogenous zero order states to the vector beams of the first order by adding in the ARCoptix. By being able to change between the zero and first order easily, relationships between the states can be found. This assists in understanding how the vector beam polarization states behave in comparison to the more familiar zero order states. The ARCoptix is a very useful device in generating an analyzing the higher order Poincaré sphere coordinates, and this system created for mapping the Poincaré sphere will be very crucial for understanding the behavior of the spatial variant vector beams.
CHAPTER 13

FUTURE WORK

The system generated in this thesis shows a simple yet robust manipulation of polarization states. The vector beam polarization states found on the first order Poincaré spheres are still recent developments in optics. The future work for this experiment would be to continue understanding the idea of the states on the first order Poincaré sphere. This system showed how an introductory optics concept of changing the phase difference and rotating the orientation of polarization state can be applied to the vector beams with a linear relationship. The ideas described in this thesis could play a large role in understanding how the first order vector beams behave when the rate of the phase change and/or rotation changes. This thesis focused on mapping the polarization states onto the Poincaré sphere, but the idea of what happens when the polarization is varied rapidly was not explored.

To begin to understand more about the higher order systems, the experimental setup could be converted to be completely voltage driven. The ARCoptix can change the polarization state from one side of the Poincaré sphere to the other, and both of the phase shifters $\phi_1$ and $\phi_2$ could also be driven by a voltage by replacing the micrometer adjusted phase shifters with voltage driven phase shifters. This improvement of the optical system would produce an even more simple system that requires less physical manipulation, and therefore less probability of causing misalignment.

Finally, another step this experiment could take would be to analyze the states above the first order. Just as the ARCoptix produces the first order from the zero order simply by passing homogenous light through it, another device could be used to generate the 2$^{nd}$ order polarization states. The same optical system could be used to map the 2$^{nd}$ order Poincaré
sphere, but I believe this experiment would be preceded by a larger understanding of the first order polarization states.
REFERENCES


APPENDIX A

Stokes Parameters and the Higher Order Spheres

All of the polarization states used in this experiment can be described by the Jones Vectors, but the more common representation is using Stokes Parameters. Stokes parameters also uses vector notation, but consists of 4 coordinates. The Stokes parameters are important because they consist of all real numbers and analysis the intensity of the beam. When performing experiments where the intensity is important, Stokes parameters are preferred. I used Jones vectors because the wave plates were analyzed with Jones vectors and I needed to ensure that the proper polarization state is produced.

The Stokes parameters can be described using the idea of 4 difference filters, the first transmits all of the light through, the second is a horizontal linear polarizer, and the third is a +\(\pi/4\)° linear polarizer. The fourth filter is a circular polarizer. The transmitted irradiances are \(I_0, I_1, I_2, I_3\) respectively. The Stokes parameters are defined as such\(^8\)

\[
S_0 = 2I_0, \quad (A.1)
\]
\[
S_1 = 2I_1 - 2I_0, \quad (A.2)
\]
\[
S_2 = 2I_2 - 2I_0, \quad (A.3)
\]
\[
S_3 = 2I_3 - 2I_0. \quad (A.4)
\]

\(S_0\) is simply the incident irradiance and \(S_1, S_2,\) and \(S_3\) are the irradiances of the difference polarized beams which in turn specify if the beam is polarized in a certain direction. In the case of a beam that isn’t polarized in a certain state, but is partially polarized, the degree of polarization can be found such that:
\[ V = \frac{(S_1^2 + S_2^2 + S_3^2)^{\frac{1}{2}}}{S_0} \]  \hspace{1cm} (A.5)

In general, the basis of the Stokes parameters is normalized with respect to \( S_0 \) such that the final states are:

- **Horizontal**:\[ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \] \hspace{1cm} (A.6)

- **Vertical**:\[ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \] \hspace{1cm} (A.7)

- **\(+\pi/4\) State**:\[ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \] \hspace{1cm} (A.8)

- **\(-\pi/4\) State**:\[ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \] \hspace{1cm} (A.9)

- **Right Circular**:\[ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \] \hspace{1cm} (A.10)

- **Left Circular**:\[ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \] \hspace{1cm} (A.11)
The equations above are the Stokes parameters for the zero order coordinates of the Poincaré sphere, but the higher orders can be described as well.

Recall from Chapter 2 that the higher order Poincaré spheres are generated by the azimuthal phase factor $e^{\pm il\phi}$. The $l$ determines the order of the angular momentum. There can now be a more general representation of the polarization states in terms of $l$. Every polarization on the Poincaré sphere can be achieved by a linear combination of circular polarizations, so they are the first to be mapped.\(^{11}\)

\[
|\mathbf{R}_l\rangle = \frac{e^{-il\phi}(\hat{x} + i\hat{y})}{\sqrt{2}}
\]  
(A.12)

\[
|\mathbf{L}_l\rangle = \frac{e^{il\phi}(\hat{x} - i\hat{y})}{\sqrt{2}}
\]  
(A.13)

The entire beam is the linear combination of the two such that:

\[
|\psi_l\rangle = \psi_R^l |\mathbf{R}_l\rangle + \psi_L^l |\mathbf{L}_l\rangle
\]  
(A.14)

The azimuthal phase factor reduces to one when there is no angular momentum. The other coordinates around the equator can be found using the right and left circular such that horizontal is the normalized sum of the two and vertical is the complex normalized difference.

\[
|\mathbf{H}_l\rangle = \frac{1}{2}(|\mathbf{R}_l\rangle + |\mathbf{L}_l\rangle) = -\sin(l\phi)\hat{x} + \cos(l\phi)\hat{y}
\]  
(A.15)

\[
|\mathbf{V}_l\rangle = \frac{-i}{2}(|\mathbf{R}_l\rangle - |\mathbf{L}_l\rangle) = \cos(l\phi)\hat{x} + \sin(l\phi)\hat{y}
\]  
(A.16)

The same can be done with the diagonal and antidiagonal where they are the sum and difference of the horizontal vertical respectively, but they are not needed to achieve the higher order Stokes parameters. Only the circular polarizations, which are the basis for the entire sphere are needed to derive the Stokes parameters.

\[
S'_0 = |\langle \mathbf{R}_l | \psi_l \rangle|^2 + |\langle \mathbf{L}_l | \psi_l \rangle|^2 = |\psi_R^l|^2 + |\psi_L^l|^2
\]  
(A.17)
\[ S_1' = 2 \text{Re} \left( \langle R_i \mid \psi_i' \rangle^\ast \langle L_i \mid \psi_i' \rangle \right) = 2 \| \psi_{R_i}' \| \| \psi_{L_i}' \| \cos \phi \]  
(A.18)

\[ S_2' = 2 \text{Im} \left( \langle R_i \mid \psi_i' \rangle^\ast \langle L_i \mid \psi_i' \rangle \right) = 2 \| \psi_{R_i}' \| \| \psi_{L_i}' \| \sin \phi \]  
(A.19)

\[ S_3' = \left| \langle R_i \mid \psi_i' \rangle \right|^2 - \left| \langle L_i \mid \psi_i' \rangle \right|^2 = \| \psi_{R_i}' \|^2 - \| \psi_{L_i}' \|^2 \]  
(A.20)

Where \( \phi = \arg(\psi_{R_i}') - \arg(\psi_{L_i}') \) and \( \| \psi_{R_i}' \|^2, \| \psi_{L_i}' \|^2 \) are the intensities of the circular polarization states.

The Stokes parameters are important when quantitative analysis is needed. The Stokes parameters analyze the intensity of the polarized beam as well as the degree of polarization, which are all real numbers. This allows for a concrete analysis of the output and how when it is polarized in its designated state.
APPENDIX B

CALIBRATION PHASE DIFFERENCE OF VARIABLE WAVEPLATE

A variable waveplate can be used to change the phase based on the thickness of the waveplate. Each variable waveplate can be calibrated to find each thickness that corresponds to different phases. The phase for a variable waveplates is described as:

\[ \phi = \frac{2\pi}{\lambda} d(n_e - n_o), \]  

(B.1)

where \( d \) is the thickness of the wave plate. In a variable retarder \( d \) can be varied either mechanically or electrically through applying a voltage. For simplicity a mechanically controlled variable wave plate called a Soleil-Babinet compensator is used for this experiment, but an electronically variable retarder could also be applicable and possibly more effective in produce results. A Soleil-Babinet compensator consists of two wedges of quartz on top of a plane-parallel slab. The total thickness of the quartz is varied by moving the position of the wedges by changing the position of a micrometer which reads off the entire thickness of the quartz. This thickness difference causes different phase differences and can create a waveplate for any desired phase difference.

Analysis can be done using a circular polarization analyzer to determine what micrometer reading turns the compensator into a quarter wave plate and any other wave plate. The waveplate phase can be defined by

\[ \varphi_1 = \frac{2\pi}{\lambda} d(n_e - n_o) = kD, \]  

(B.2)

where \( k \) is the scaling constant and \( D \) is the micrometer distance. To find the appropriate micrometer measurements for a variable wave plate, it is placed between crossed polarizers. When there is no phase shift, or when the variable waveplate is a full waveplate then the intensity will be zero because crossed polarizers emit no light. A half waveplate causes a \( \pi/2 \)
rotation which will correspond to the highest intensity because all the light was rotated to the second polarizers’ orientation and all pass through. By plotting the intensity, the micrometer readings for different corresponding waveplates can be created and an average value for the scaling constant can be found. For this experiment, the scaling constant is not as important as knowing what micrometer measurements correspond to the phase difference.

Table B.1: Micrometer measurements (D) for different relative phase difference in a Soleil-Babinet Compensator.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Intensity($I/I_o$)</th>
<th>D(mm)</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>1</td>
<td>5</td>
<td>628.3</td>
</tr>
<tr>
<td>$2\pi$</td>
<td>0</td>
<td>11.42</td>
<td>550.2</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>.5</td>
<td>2</td>
<td>785.4</td>
</tr>
<tr>
<td>$5\pi/2$</td>
<td>1</td>
<td>18</td>
<td>436.3</td>
</tr>
<tr>
<td>$3 \pi/2$</td>
<td>.5</td>
<td>8</td>
<td>589.0</td>
</tr>
</tbody>
</table>

Figure B.1: Intensity distribution versus micrometer measurements from Soleil-Babinet Compensator.

Each variable waveplate is manufactured differently and needs to undergo this calibration if the micrometer measurements for the phase are desired.
APPENDIX C

HALF MOON CIRCULAR POLARIZATION STATES

As stated in Chapter 8, the voltage can be applied to the phase compensator to homogenize the beam so that $V_1=1.4V$. At this voltage setting it was possible to remove the dark line in the center as much as possible. Proper homogenization was difficult to achieve, but the intensity of the dark line could be minimized. I started with the equatorial points as they were the easiest to measure with a linear analyzer. The images of the four can be seen below.

C.1 Case 1: $\varphi_2 = 0$ Produces Radial Polarization State

For radial polarization state produced when $\varphi_2 = 0$, the expected results is that vertical analyzer (b) illuminates the upper and lower sections, horizontal analyzer (d) illuminates the horizontal components, and the spirals illuminate $+\pi/4$ (c) and $-\pi/4$ (e) for the $+\pi/4$ spiral and $-\pi/4$ spiral respectively. Circular (f), (g) should not be affected. So far the expected results are produced in Figure C.1.

![Figure C.1: Radial polarization through linear and circular analyzers.](image)

C.2 Case 2: $\varphi_2 = \pi/2$ Produces $\pi/4$ Spiral Polarization State

The next state around the equator is the $-\pi/4$ spiral that occurs at $\varphi_2 = \pi/2$. As a rule of thumb for the higher orders, the state that is being produced will cause vertical illumination for the analyzer of the same state. Therefore, for the $-\pi/4$ spiral above, the $-\pi/4$
slanted analyzer should produce bright spots in the vertical plane. This also means the opposite polarization, +π/4 will be on the horizontal plane. Both are exhibited above and it appears to follow the same behavior. The −π/4 analyzer (e) illuminates the y axis, and the +π/4 analyzer (c) illuminates the x axis. Vertically polarized light (b) is on the +π/4 axis, and horizontally polarized light (d) is on the −π/4 axis. The circular seems to be behaving strangely for the right circular (g) in this case, but could be due to the same complications as the zero order with improper alignment.

Figure C.2: Negative π/4 radians spiral polarization through linear and circular analyzers.

C.3 Case 3: φ₂ = π Produces Azimuthal Polarization State

For the azimuthal polarization state produced when φ₂ = π, the horizontal analyzer (d) should create bright vertical sections, and the vertical analyzer (b) should illuminate the horizontal axis of the beam. The results in Figure C.3 are expected.

Figure C.3: Azimuthal polarization through linear and circular analyzers.

C.4 Case 4: φ₂ = 3π/2 Produces +π/4 Spiral Polarization State

When φ₂ = 3π/2 the +π/4 spiral vector beam is produced. The expected results for the +π/4 spiral will be the reverse of the −π/4, where the −π/4 analyzer (e) illuminates the horizontal axis and the +π/4 analyzer (c) illuminates the vertical axis. All of the rotator
measurements behaved as expected. The state after this returns to radial polarization indicating a complete loop around the equator.

Figure C.4: Positive \( \pi/4 \) radians spiral polarization through linear and circular analyzers.

After completing the rotator measurements I began to change the phase shift \( \phi_1 \) to produce circularly polarized light starting from radial polarization.

**C.5 Case 5: \( \phi_1 = 3\pi/2 \) Produces Left Circular Polarization State**

The results when \( \phi_1 = 3\pi/2 \) are not the expected results. The circular polarization states (f), (g) should be the same throughout the beam, with the vortex in the middle like the previous results display. The results are showing a half and half polarization state for the circular analyzers with left circular on the top (f), and right circular on the top (g), as well as the dark line I had removed in the previous photos. I got the same results for the right circular pole as well with the states flipped. The coordinates around the equator did not have any complications, so I can determine that it is a problem in the interaction of the phase of the polarization states.

Figure C.5: Left circular polarization through linear and circular analyzers with some complication in the phase.

The states are produced because setting the voltage on the ARCoptix tries to correct the half wave plate twist in the lower half as stated in Chapter 8. The phase compensator
adds in a voltage driven half-wave plate in the lower half of the cell. When the circular beam enters the ARCoptix the upper half is not changed, but the lower half is flipped to the opposite polarization state. If a left circular beam enters, the results will be half left circular, half right circular, which can be seen above in Case 5 for the circular analyzers (f), (g). Due to the results in Figure C.5, I did not set $V_1$ to 1.4V but left it at 0.00V to produce the actual results found in this thesis. The half-moon states are interesting, and it would be an experiment in its own right to understand more about this case, but they are not the desired polarization states because they cannot be mapped to a Poincaré sphere.