APPLICATION OF DYNAMIC LINEAR REGRESSION MODEL &
HYPOTHESIS-TESTING FOR DAMAGE DETECTION OF HIGHWAY
BRIDGE

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Application of Dynamic Linear Regression Model & Hypothesis-Testing for Damage

Detection of Highway Bridge

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ABSTRACT OF THE THESIS

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As a result of aging under environmental conditions, highway bridges are gradually deteriorating. This issue has gradually become a worldwide concern. Vibration-based structural damage detection has attracted more and more attention over the past decade. However, the effects of changes in environmental conditions have been recognized as main barriers against the application of vibration based damage identification techniques for real-world bridges.

In this thesis a statistical method is presented for highway bridge damage detection by applying hypothesis-testing theory in conjunction with dynamic linear regression model. This method makes improvement by considering the environmental effects in damage detection. Thus changes of indicator caused by damage can be separated from those caused by environmental conditions. As a result, the use of this method can avoid misclassification for damage detection results.

In order to account for effects of changes in environmental conditions on structural damage detection results, a statistical damage detection procedure is presented. It is assumed that the dynamic responses from both healthy (undamaged) state and damaged state are available. The presented method consists of (1) applying a dynamic linear regression model, auto-regressive with external inputs, to model the relationship between damage signatures and time varying environmental conditions, and (2) performing damage detection based on hypothesis-testing. The effectiveness and robustness of the presented procedure is demonstrated by damage detection of a simply supported beam with different level of damage severity.

The research work presented in this thesis contributes to apply statistical techniques for the development of robust and reliable vibration-based structural health monitoring systems for real-world structures.
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CHAPTER 1

INTRODUCTION

Due to ageing under varying environmental conditions, highway bridges undergo progressive deterioration, which has become a worldwide concern. For example, in the United States, over 50% of all bridges were built before the 1940’s and approximately 27.5% of these structures are structurally deficient (He, 2008; Stalling, Tedesco, El-Mihilmy, & McCauley, 2000).

A traditional method called visual inspection has been widely applied for damage detection. However, this method has some inherent drawbacks. One of them is that it is very hard to detect the damages hiding inside the structure, i.e., the damage can only be detected when it becomes severe enough and is thus visible. Thus the vibration-based structural damage detection and bridge health monitoring have attracted more and more attention in recent years (Doebling, Farrar, Prime, & Shevitz, 1996; Farrar & Jauregui, 1998; Guan, 2006; Sohn et al., 2003). This method is based on the fact that the structural modal properties (i.e., natural frequencies, damping ratios, and mode shapes of structures) are a function of structural damage. Therefore, the structural damage can be detected by changes in these modal parameters.

In the area of damage detection and health monitoring of real-world bridges, the environmental effects on the structural modal parameters have been recognized as one of the main barriers. Changes in structural modal parameters can be caused by structural damage (e.g., loss of structural stiffness) and/or varying environmental conditions (e.g., temperature, wind, and humidity). In previous limited studies, it has been shown that the effects of changes in the environmental conditions on the changes in the structural modal parameters are very significant; they may be even larger than those caused by structural damage (Abdel Wahab & De Roeck, 1997; Cornwell, Farrar, Doebling, & Sohn, 1999; Peeters & De Roeck, 2001; Xia, Hao, Zanardo, & Deeks, 2006).

In this thesis, a damage detection procedure based on a dynamic linear regression model and hypothesis-testing is presented for bridge damage detection, assuming that the
structure modal parameters of the bridge from both healthy (undamaged) state and damaged state are available. The method presented includes (1) modeling the relationship between natural frequencies and environmental conditions such as temperature using a dynamic linear regression model to account for the environmental effects on the structural behaviors and (2) damage detection based on hypothesis-testing. The effectiveness and robustness of the presented procedure is demonstrated by damage detection of a numerical simply supported beam with different level of damage severity.

The research work presented in this thesis contributes to the development of robust and reliable vibration-based structural health monitoring systems for real-world structures.
CHAPTER 2

DYNAMIC LINEAR REGRESSION MODEL

2.1 INTRODUCTION

Former studies showing that the variations in natural frequencies caused by changes in environmental conditions are very significant; they may even induce changes larger than those caused by structural damage (e.g., Abdel Wahab & De Roeck, 1997; Cornwell et al., 1999; Peeters & De Roeck, 2001). Thus in order to obtain reliable damage detection results, it is essential to account for the effects of changes in environmental conditions on the structural modal parameters. Usually, the effects of varying temperature, compared to other environmental parameters such as humidity and wind, on the bridge structural modal parameters are most significant (He, 2008; Peeters & De Roeck, 2001). In this thesis, the natural frequency of bridges is considered as damage signature/indicator while the temperature is used to represent the varying environmental conditions. To model the relationship between natural frequency and temperature, a dynamic linear regression model is discussed. The methodology presented here is also applicable to account for effects of other environmental conditions on structural modal parameters.

2.2 DYNAMIC LINEAR REGRESSION MODEL

For a single-input-single-output (SISO) linear, time-invariant system, probably the simplest input-output relationship is obtained by describing it as a linear difference equation (Ljung, 1999)

\[ y(t) + a_1 y(t-1) + \cdots + a_n y(t-n) = b_1 u(t-n_0) + b_2 u(t-1-n_0) + \cdots + b_z u(t-n_z-n_0+1) + e(t) \]

with

- \( y(t) \) = output (e.g., an identified natural frequency)
- \( u(t) \) = input (e.g., a measured temperature)
\( n_1 = \) auto-regressive order
\( n_2 = \) exogeneous order
\( n_0 = \) pure time delay between input and output
\( a_i, b_j = \) model parameters
\( e(t) = \) error term

\[ (2.1) \]

The error term is assumed to be zero-mean white noise with covariance

\[ E[e(t)e(t-t)] = l d(t) \]

with

\[ d(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{others} \end{cases} \]

\[ E[\cdot] = \text{expected value operator} \]

This well known model Error! Reference source not found. is called an ARX model, where AR refers to the autoregressive part and X to the extra input (called the exogeneous variable in econometrics). It can be also called dynamic linear regression (DLR) model. The orders \( n_1 \) and \( n_2 \) determine the number of model parameters: \( a_i (i = 1, \cdots, n_1) \) and \( b_j (j = 1, \cdots, n_2) \). By introducing the backward shift operator \( q^{-1} : q^{-1} y(t) = y(t-1) \), the model Error! Reference source not found. is rewritten as (Ljung, 1999)

\[ A(q)y(t) = B(q)u(t) + e(t) \]

with

\[ A(q) = 1 + a_1 q^{-1} + \cdots + a_{n_1} q^{-n_1} \]

\[ B(q) = b_0 q^{-n_0} + b_1 q^{-n_0-1} + \cdots + b_{n_2} q^{-n_0-n_2+1} \]

\[ (2.3) \]

The DLR model in equation (2.1) can be rewritten as

\[ y(t) = j^T(t)q + e(t) \]

with

\[ \theta = \text{model parameters vector and} \]

\[ \theta^T = [a_i \cdots a_{n_1} b_1 \cdots b_{n_2}]_{n_1+n_2} \]

The model parameters \( \theta \) of a DLR model can be estimated by a linear least squares method as following, which makes the DLR model popular in practical application. Let

\[ j^T(t) = \left[ \begin{array}{cccc} y(t-1) & \cdots & y(t-n_1) & u(t-n_0) & \cdots & u(t-n_2-n_0+1) \end{array} \right]_{n_1+n_2} \]

\[ (2.5) \]
Then the parameter $\theta$ can be estimated as

$$\hat{\theta} = \sum_{j=1}^{N} j (t) y(t) \left( \sum_{j=n_1}^{n_2} j y(t) \right)$$

with

$$N = \text{Number of data points}$$

With different choice of $n_1, n_2$ and $n_0$, different DLR models can be identified based on the input and output data. The loss function is usually used as a criterion to assess the quality of these different identified DLR models. The loss function is defined as (Ljung, 1999; Peeters & De Roeck, 2001)

$$\hat{\ell} = \frac{1}{N} \sum_{t=1}^{N} e^2(t, \hat{\theta})$$

with

$$e(t, \hat{\theta}) = y(t) - y(t, \hat{\theta})$$

The loss function is also an estimate of the noise covariance $\ell$. In practical applications, the loss function usually keeps decreasing as the model order increases. Therefore some other criteria such as Akaike's information criterion (AIC) and Rissanen's minimum description length (MDL) criterion are necessary to estimate the system orders (Ljung, 1999).

### 2.3 A Numerical Example

Now let us consider a simple example, as shown in Figure 2.1, to illustrate the application of DLR model to predict the natural frequency with varying temperatures.

**Figure 2.1. Single degree-of-freedom system.**
For a single-degree-freedom as shown in Figure 2.1, the natural frequency is easy to be obtained

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

with

\[ m = \text{mass of the system} \]
\[ k = \text{stiffness of the system} \]

The stiffness \( k \) is a function of temperature resulting in changes of the natural frequency with varying temperature. In this example the mass is assumed to be \( m = 1 \). Based on research work performed in structural health monitoring such as (Xu & Wu, 2007; He 2008), the stiffness is simulated as:

\[ k = k_0(1 - k_1 W) \]

with

\[ k_0 = 64p^2, \quad k_1 = 0.05 \]

\[ W = \text{Temperature} \]

The temperature \((W)\) is simulated in the following way:

\[ W = 20 + at + 2\sin(w_1 t) + 2\sin(w_2 t) \]

with

\[ w_1 = p/12, \quad w_2 = p/48 \]
\[ a = 0.04 \]
\[ t = \text{time} \]

Figure 2.2 shows simulated temperature measurements during a period of 200 hours. Correspondingly, the natural frequency is simulated during these 200 hours (see Figure 2.3).

In order to account for the measurement noise and system identification errors, zero-mean Gaussian white noise with 20% noise level is added to the simulated natural frequency. The noise level is defined as the ratio (in percent) of the root mean square (RMS) of the added noise process to the RMS of the simulated natural frequency.

Based on the temperature measurements and natural frequency in this numerical example, a DLR model is identified to represent the natural frequency as a function of measured temperature. In this example, the input is the temperature and the output is the natural frequency. In the identification of DLR model, the system order is estimated \( s n_1 = 4 \).
Figure 2.2. Simulated temperature measurements during a period of 200 hours.

Figure 2.3. Simulated natural frequency during a period of 200 hours.
n2 = 2, and n0 = 1, based on the AIC criterion. Another quality criterion to assess the quality of the identified DLR model is to investigate the auto-correlation function of its prediction error due to the fact that the prediction error should be zero-mean white noise in the case that a good DLR model is obtained. The auto-correlation function of the is estimated as (Peeters & De Roeck, 2001)

\[ \hat{R}_e(t) = \frac{1}{N} \sum_{k=1}^{N} e(t, \hat{\theta}) e(t - \tau, \hat{\theta}) \]  

2.11)

The auto-correlation functions of prediction errors for the identified DLR model is plotted in Figure 2.4, together with the 99% confidence intervals. Once a model representing the natural frequency as a function of measured temperature is obtained, it can be used to predict the natural frequency based on the measured temperature (fresh data). As an illustration, Figure 2.5 shows the predicted natural frequency using the identified DLR model based on the measured temperature (simulated). From this figure, it can be concluded that the identified DLR model fit the natural frequency and temperature in this numerical example very well.

Figure 2.4. Normalized auto-correlation of the prediction errors of the identified DLR model.
**Figure 2.5.** Comparison of the estimated natural frequencies using identified DLR model and its counterparts used to identify DLR model: dashed line, simulated natural frequency based on temperature measurements; dot, natural frequency used to identify DLR model.

### 2.4 SUMMARY

In previous studies, the variations in modal properties due to changes in environmental conditions have been shown to be very significant. In this chapter, a dynamic linear regression model is presented to correlate natural frequency with temperature. A numerical example is used to demonstrate its application, based on which, it is found that the natural frequency can be described as a function of temperature by using the presented linear dynamic regression model.
CHAPTER 3

HYPOTHESIS-TESTING

3.1 INTRODUCTION

A linear dynamic regression model has been described in Chapter 2 to account for the effects of changes in environmental conditions on bridge behaviors. It is assumed that the dynamic properties of the bridge such as natural frequencies and measured environmental conditions such as temperature are available from both undamaged and damaged states of the structure. This chapter will describe how to detect damage by using hypothesis-testing.

3.2 HYPOTHESIS-TESTING

For sake of convenience, the natural frequencies and temperature measured from undamaged state of the bridge are denoted as $f^u, W^u$, respectively. Their counterparts from the unknown (damaged) state are denoted as $f^d, W^d$, respectively. Suppose that the DLR $(n_1, n_2, n_0)$ has been identified to describe the natural frequency as a function of temperature. Thus based on the measured temperature, the natural frequency can be estimated. First, the identified DLR model is used to estimate the natural frequency at the undamaged state based on the measured temperature and obtain the estimation error term $e^u(t)$:

$$e^u(t) = f^u(t) - \hat{f}^u(t)$$

with

$$\hat{f}^u = G(q, \hat{\theta})W^u$$

$$G(q, \hat{\theta}) = \frac{B(q, \hat{\theta})}{A(q, \hat{\theta})}$$

$$(3.1)$$

$$A(q, \hat{\theta}) = 1 + \hat{a}_1q^{-1} + \cdots + \hat{a}_nq^{-n_1}$$

$$B(q, \hat{\theta}) = \hat{b}_1q^{-n_0} + \hat{b}_2q^{-n_0-1} + \cdots + \hat{b}_{n_2}q^{-n_0-n_2+1}$$

$q^{-1} = \text{backward shift operator, i.e., } q^{-1}W^u(t) = W^u(t-1)$

Then the DLR $(n_1, n_2, n_0)$ model described above is used to estimate the natural frequency of the damaged state based on the measured temperature and obtain the corresponding
estimation error term $e^d(t)$:

$$e^d(t) = f^d(t) - \hat{f}^d(t)$$

with

$$\hat{f}^d = G(q, \hat{\Theta})T^d$$

$$G(q, \hat{\Theta}) = \frac{B(q, \hat{\Theta})}{A(q, \hat{\Theta})}$$

$$A(q, \hat{\Theta}) = 1 + \hat{a}_1q^{-1} + \cdots + \hat{a}_nq^{-n_1}$$

$$B(q, \hat{\Theta}) = \hat{b}_1q^{-n_0} + \hat{b}_2q^{-n_0-1} + \cdots + \hat{b}_nq^{-n_0-n_1+1}$$

$q^{-1}$ = backward shift operator, i.e., $q^{-1}W^d(t) = W^d(t-1),$

It should be noted that the residual term $e^d(t)$ includes the part of the output that the DLR $(n_1, n_2, n_0)$ could not reproduce. Therefore, if $f^d$ is from the same system as that of $f^u$, in other words, if $f^d$ also comes from the undamaged state of the bridge, the DLR $(n_1, n_2, n_0)$ which is used to model relationship between $f^u$ and $W^u$ will be able to model the relationship between $f^d$ and $W^d$. The Two error terms $e^u(t)$ and $e^d(t)$ will be similar. In contrary, if $f^d$ is from a damaged state of the bridge (a state different from undamaged state), then the error term $e^d(t)$ will be significant different from error term $e^u(t)$.

Thus, a statistical damage diagnosis procedure is proposed based on hypothesis – testing theory. Two error terms $e^u(t)$ and $e^d(t)$ are considered as two random variables with normal distribution $e^u(t) \sim N(m^u, s^u)$ and $e^d(t) \sim N(m^d, s^d)$, respectively. Both the mean and standard deviation of these two random variables are unknown. Since two residual terms $e^d(t)$ and $e^u(t)$ will be similar if the $f^d$ and $f^u$ are from the same state of the bridge (undamaged state), the following hypothesis – testing is proposed:

$$H_0 : m^d - m^u = 0$$

$$H_1 : m^d - m^u < 0$$

(3.3)

The one side testing is used herein due to the fact that the natural frequency will be reduced by structural damage. The mean value of error term $m^d$ will be smaller than $m^u$ when damage exists.
The test statistic is given by

\[
T = \frac{(\bar{e}_d - \bar{e}_u) - (m' - m'')}{\sqrt{\frac{s^2_d}{n_d} + \frac{s^2_u}{n_u}}}
\]

with

\[
\bar{e}_d = \frac{1}{n_d} \sum_{k=1}^{n_d} e'_d; \quad \bar{e}_u = \frac{1}{n_u} \sum_{k=1}^{n_u} e''_u; \quad (3.4)
\]

\[
s^2_d = \frac{1}{n_d - 1} \sum_{k=1}^{n_d} (e'_d - \bar{e}_d)(e'_d - \bar{e}_d)
\]

\[
s^2_u = \frac{1}{n_u - 1} \sum_{k=1}^{n_u} (e''_u - \bar{e}_u)(e''_u - \bar{e}_u)
\]

The corresponding rejection is defined as

\[
\{T < - t_{\alpha} (df)\}
\]

with

\[
df = \frac{(n_1 - 1)(n_2 - 1)}{(n_2 - 1)c^2 + (n_1 - 1)(1 - c)^2}
\]

\[
c = \frac{\left(\frac{s^2_1}{n_1}\right)\left(\frac{s^2_2}{n_2}\right)}{\left(\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}\right)}
\]

In the above equations, \(n_d\) and \(n_u\) are the data points of the \(e'_d (t)\) and \(e''_u (t)\), respectively; \(t_{\alpha}\) is such that \(P(T < - t_{\alpha}) = \alpha\) for a \(t\) distribution with \(df\) degrees of freedom. The Satterthwaite's approximation (Fleiss, 1986) is used to estimate the degree of freedom in above equations. In the case that the value \(T\) falls in the rejection region, then the null hypothesis will be rejected at the \(\alpha\) level of significance and it can be concluded that the bridge is damaged \((m' - m'' < 0)\). In practice, \(\alpha\) is usually taken as 5%, which indicates that the probability of rejecting \(H_0\) (classifying bridge as damaged) when \(H_0\) is true (bridge is not damaged) is 5%. It should be noted that the effect of damage severity (with different level loss of stiffness) on the damage detection results using hypothesis-testing procedure in this study is presented in Chapter 4.
3.3 SUMMARY

Following Chapter 2, a damage detection procedure based on hypothesis – testing theory is described in this chapter. In next chapter, the effectiveness and robustness of presented statistical damage detection method in this thesis (Chapter 2 and Chapter 3) will be demonstrated from a numerical example.
CHAPTER 4

HYPOTHESIS-TESTING

4.1 INTRODUCTION

In this Chapter, numerical cases study is performed to demonstrate the procedure of damage detection using the presented statistical damage detection method in Chapter 2 and 3. Meanwhile the effectiveness and robustness of presented statistical damage detection method will be also shown through this numerical case study.

4.2 NUMERICAL CASES STUDY

The numerical case study is performed based on a simply supported beam as described in the following section.

4.2.1 Numerical Beam Model

A simple span girder bridge with span length of 10m is used in the numerical case study. The material properties for the beam are: modulus of elasticity $E_0 = 3 \times 10^{10} Pa$ and mass density $\rho = 2400 kg/m^3$. The modulus of elasticity is function of temperature which is simulated as,

$$E = E_0 (1 - k_1 W)$$

with

$$k_1 = 0.1$$

$W = \text{Temperature}$

The beam is assumed to have a uniform rectangular cross section with 0.2m height and 0.2m width. Figure 4.1 shows the layout of the beam and finite element mesh, which consists of 11 nodes and 10 elements. In this study, the 6th and 7th elements are assumed to be damaged by reduction of their modulus of elasticity. Three different reduction ratios of modulus of elasticity for these two elements, 5% (Case 1), 7.5% (Case 2), and 10% (Case 3) are considered. Table 4.1 lists the natural frequencies of the first four vibration modes for
both undamaged and damaged beam. Changes in natural frequencies can be found from this table. It should be noted, in the practical application, the changes in environmental conditions will also cause changes in natural frequencies. Therefore, in order to use changes in natural frequency as damage indicator, the presented statistical damage detection method has to be applied. As an illustration, Figure 4.2 shows the mode shapes of the first four vibration modes for the undamaged beam.

<table>
<thead>
<tr>
<th>Mode No</th>
<th>Undamaged State</th>
<th>Damaged State</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Case 1 (5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case 2 (7.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case 3 (10%)</td>
</tr>
<tr>
<td>1</td>
<td>3.21</td>
<td>3.18</td>
</tr>
<tr>
<td>2</td>
<td>12.83</td>
<td>12.78</td>
</tr>
<tr>
<td>3</td>
<td>28.87</td>
<td>28.75</td>
</tr>
<tr>
<td>4</td>
<td>51.39</td>
<td>51.08</td>
</tr>
</tbody>
</table>

Figure 4.2. Mode shapes of the first four vibration modes.
4.2.2 Natural Frequencies and Temperature Measurements

It is assumed that the environmental temperature measurements are available, which is simulated as:

\[ W = 20 + \sin(w_1 t) + \sin(w_2 t) + 2\sin(w_3 t) \], with

\[ w_1 = p / 12 \]
\[ w_2 = p / 48 \]
\[ w_3 = p / 900 \]  

(4.2)

As an illustration, Figure 4.3 presents the simulated temperature. It is also assumed that natural frequencies from both undamaged state and damaged states of the beam are available. Figures 4.4, 4.5, 4.6, and 4.7 show the natural frequencies of the first four vibration modes for the undamaged and damaged states.

Figure 4.3. Simulated temperature measurements.
Figure 4.4. Natural frequencies of the simply supported beam in undamaged state.

Figure 4.5. Natural frequencies of the simply supported beam in damaged state Case 1.
Figure 4.6. Natural frequencies of the simply supported beam in damaged state Case 2.

Figure 4.7. Natural frequencies of the simply supported beam in damaged state Case 3.


4.2.3 Damage Detection

In the undamaged state, the available natural frequency and temperature data are first used to identify the dynamic linear regression model to describe the relationship between natural frequency and temperature. Following the method and procedure described in Chapter 2, four DLR models are identified, corresponding to four vibration modes. Table 4.2 lists these four identified DLR models. Then these identified DLR models are applied to estimate the natural frequencies based on temperatures of three damage states. The damage detection procedure is summarized as following:

Step 1: Based on the simulated natural frequency and its corresponding estimate of the undamaged state of the beam, to obtain the error term $e^u(t)$. Then the estimate of its mean value and variance (or standard deviation) can be determined using equation (3.4). Table 4.3 list the estimate of mean value and standard deviation of the $e^u(t)$ for all four vibration modes;

Step 2: Based on the simulated natural frequency and its corresponding estimate of the damaged state (or unknown state) of the beam, to obtain the error term $e^d(t)$. Then the estimate of its mean value and variance (or standard deviation) can be determined using equation (3.4). The estimate of mean value and standard deviation of the $e^d(t)$ for all three damaged cases and four vibration modes is also listed in the Table 4.3 together with their counterparts of the undamaged state;

Step 3: To calculate the test statistic using equation (3.4), which is rewritten here

$$T = \frac{\hat{\epsilon}_e - \hat{\epsilon}_n}{\sqrt{\frac{s^2_d}{n_d} + \frac{s^2_u}{n_u}}}$$

(4.3)

Step 4: To calculate the rejection region $\{T < - t_{n-1} \}$ with $a = 0.05$ level of significance.

The test statistic and rejection region for all cases studied in this thesis is listed in Table 4.4. Figure 4.8 shows these test statistics in a bar plot. From Table 4.4 and Figure 4.8, it is found that (1) The test statistic value obtained from all three damage cases are less than $- t_{0.05}$. In another word, the test statistic values fall in the rejection region, which indicates that the bridge is damaged. (2) The test statistic value decreases with increasing the level of
Table 4.2. Four Identified DLR Models

<table>
<thead>
<tr>
<th>Model</th>
<th>DLR Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \hat{f}(t) = -0.146W(t) + 0.055\hat{f}(t-1) + 0.012\hat{f}(t-2) + 0.053\hat{f}(t-3) + 5.733 )</td>
</tr>
<tr>
<td>2</td>
<td>( \hat{f}(t) = -0.629W(t) + 0.007\hat{f}(t-1) + 0.042\hat{f}(t-2) - 0.025\hat{f}(t-3) + 25.065 )</td>
</tr>
<tr>
<td>3</td>
<td>( \hat{f}(t) = -1.404W(t) + 0.018\hat{f}(t-1) - 0.007\hat{f}(t-2) + 0.014\hat{f}(t-3) + 56.146 )</td>
</tr>
<tr>
<td>4</td>
<td>( \hat{f}(t) = -2.407W(t) + 0.057\hat{f}(t-1) + 0.018\hat{f}(t-2) + 0.013\hat{f}(t-3) + 94.822 )</td>
</tr>
</tbody>
</table>

Table 4.3. Estimate of Mean Value and Standard Deviation of Error Terms

<table>
<thead>
<tr>
<th>Mode No</th>
<th>Undamaged Case</th>
<th>Damaged Case 1</th>
<th>Damaged Case 2</th>
<th>Damaged Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \bar{e}_u )</td>
<td>( s_u )</td>
<td>( \bar{e}_d^1 )</td>
<td>( s_d^1 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.1393</td>
<td>-0.0285</td>
<td>0.1426</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.5779</td>
<td>-0.0727</td>
<td>0.573</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.2304</td>
<td>-0.0984</td>
<td>1.2533</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>2.3565</td>
<td>-0.3867</td>
<td>2.2737</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode No</th>
<th>Undamaged Case</th>
<th>Damaged Case 1</th>
<th>Damaged Case 2</th>
<th>Damaged Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s_{d}^2 )</td>
<td>( \bar{e}_d^2 )</td>
<td>( s_{d}^2 )</td>
<td>( \bar{e}_d^3 )</td>
</tr>
<tr>
<td>1</td>
<td>0.1389</td>
<td>-0.0477</td>
<td>0.1389</td>
<td>-0.0628</td>
</tr>
<tr>
<td>2</td>
<td>0.5647</td>
<td>-0.0774</td>
<td>0.5647</td>
<td>-0.1321</td>
</tr>
<tr>
<td>3</td>
<td>1.3074</td>
<td>-0.2194</td>
<td>1.3074</td>
<td>-0.2305</td>
</tr>
<tr>
<td>4</td>
<td>2.2228</td>
<td>-0.4508</td>
<td>2.2228</td>
<td>-0.6382</td>
</tr>
</tbody>
</table>

Table 4.4. Damage Detection Results Based on Hypothesis – Testing Theory

<table>
<thead>
<tr>
<th>Mode No</th>
<th>( t_{0.95} )</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T^i_d )</td>
<td>Damaged?</td>
<td>( T^i_d )</td>
<td>Damaged?</td>
</tr>
<tr>
<td>1</td>
<td>-5.54</td>
<td>Yes</td>
<td>-9.39</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>-3.46</td>
<td>Yes</td>
<td>-5.15</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>-2.17</td>
<td>Yes</td>
<td>-6.46</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>-4.57</td>
<td>Yes</td>
<td>-7.84</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Damage. The higher level of the damage, the lower the test statistic value is obtained and the more likely that the test statistic value will fall in the rejection region. (3) The damage detection results using four different vibration mode frequencies are consistent. The damage detection procedure using the presented damage detection method is demonstrated through this numerical example. Most important, the example shows that the present method is robust and effective. The damage can be successfully detected with low level of damage (5% loss of
stiffness and corresponding frequency change is less than 1%), which has a good potential to be applied to a real-world structure for structural health monitoring under varying environmental temperature.

### 4.3 SUMMARY

In this Chapter, the application of the presented hypothesis-testing based statistical damage detection method in Chapter 2 and 3 is demonstrated by performing damage detection of a numerical simply supported beam. Three different damage cases with different level of damage are considered in the study. It is found that with increasing level of damage, the test statistic value decreases and the higher possibility of the test statistic value fall in the rejection region. Meanwhile, the damage detection results obtained using four different vibration mode frequencies are consistent. From this example, it is validated that presented method can be successfully applied for damage detection under varying environmental temperature.
CHAPTER 5

CONCLUSIONS

Vibration-based structural damage detection and health monitoring has attracted more and more attention in recent years. These methods are based on the fact that the structural modal properties (i.e., natural frequencies, damping ratios, and mode shapes) are a function of structural damage. Therefore, the structural damage can be detected by changes in these modal parameters. However, the environmental effects on the varying structural modal parameters have been recognized as one of the main barriers against the application of these modal parameters for damage detection and health monitoring of real-world bridges. Variations in modal parameters due to changes in temperature have been shown to be very significant in previous studies; they may be even larger than those caused by structural damage.

In order to account for the changes in environmental effects and inherent uncertainty in vibration based structural damage detection, a statistical damage detection procedure based on hypothesis-testing is presented. The presented method consists of two steps. A dynamic linear regression model is applied in the first step to describe the natural frequencies as a function of time varying temperatures. This model makes it possible to account for the effect of changes in environmental conditions on the modal parameters of the bridge. Then on the second step, the identified linear regression model is applied to estimate natural frequencies corresponding to all different states of the bridge. Based on the estimation errors, a null hypothesis is proposed. If the test statistic falls in the rejection region, then the null hypothesis will be rejected at the \( \alpha \) level of significance and it can be concluded that the bridge is damaged.

The effectiveness and robustness of the presented procedure is demonstrated by damage detection of a numerical simply supported beam with different level of damage. The results show that the damage detection procedure has great potential to detect damage at early stage under the varying environmental conditions and inherent uncertainties in the dynamic response measurements. The research work presented in this thesis contributes to
apply statistical techniques for the development of robust and reliable vibration-based structural health monitoring systems real-world structures.
REFERENCES


